Debye sheath / Základy teorie stěnové vrstvy

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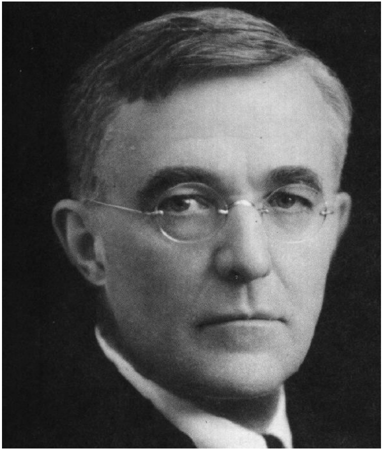
Irving Langmuir

Langmuir would never have thought to ask the question "when will I ever use this?". His scientific inquiries sprang from a curiosity that saw a purpose in everything. That practical applications flowed from his theoretical wanderings was to him the "icing on the cake".



Irving Langmuir

- 31 January 1881 – 16 August 1957.
- American chemist and physicist.
- In 1919 famous article „The Arrangement of Electrons in Atoms and Molecules“.
- General Electric (1909–1950) - He invented the gas-filled incandescent lamp, the hydrogen welding technique, improvemed the diffusion pump.
- He and colleague Lewi Tonks discovered that the lifetime of a tungsten filament was greatly lengthened by filling the bulb with an inert gas, such as argon.
- In 1932 Nobel Prize in Chemistry for his work in surface chemistry.
- In 1924 he invented the diagnostic method for measuring both temperature and density with an electrostatic probe, now called a Langmuir probe.

Irving Langmuir

- He joined Katharine B. Blodgett to study thin films and surface adsorption. They introduced the concept of a monolayer (a layer of material one molecule thick) and the two-dimensional physics which describe such a surface.
- In 1938, Langmuir's scientific interests began to turn to atmospheric science and meteorology. This research led him to theorize that the introduction of dry ice and iodide into a sufficiently moist cloud of low temperature could induce precipitation (cloud seeding).

Irving Langmuir

Cephenemyia pratti (střeček)

Říše Animalia - živočichové, kmen Arthropoda - členovci, třída Insecta - hmyz, řád Diptera - dvoukřídlí, čeleď Oestridae - střečkovití.

Fastest of all flying insects?

It was reported for many years that Cephenemyia was the fastest of all flying insects, cited by the New York Times and Guinness Book of World Records. The source of this remarkable claim was an article by entomologist Charles H. T. Townsend (x mathematical physicist John Sealy Townsend) in the 1927 Journal of the New York Entomological Society, wherein Townsend claimed to have estimated a speed of over 800 miles per hour while observing Cephenemyia pratti in New Mexico.
In 1938 Irving Langmuir, recipient of the 1932 Nobel Prize in Chemistry, examined the claim in detail and refuted the estimate. Among his specific criticisms were:

- To maintain a velocity of 800 miles per hour, the 0.3-gram fly would have had to consume more than 150% of its body weight in food every second;
- The fly would have produced an audible sonic boom.
- The supersonic fly would have been invisible to the naked eye.
- The impact trauma of such a fly colliding with a human body would resemble that of a gunshot wound.
- Using the original report as a basis, Langmuir estimated the deer botfly's true speed at 25 miles per hour.
- Langmuir estimated the fly's true speed at 25 miles per hour.

The latest edition of Encyclopaedia Britannica cites a speed of 80 km (50 mi) per hour for this fly. Time magazine published an article in 1938 "debunking"Townsend's calculations. But the New York Times, which ran a story in 1937 on "the fastest creature that lives', has not yet published a correction.

Historical Point of View

Sheaths were first described by American physicist Irving Langmuir. In 1923 [1] he wrote :

- Electrons are repelled from the negative electrode, positive ions are drawn towards it, there is a sheath of definite thickness containing only positive ions and neutral atoms.
- The thickness of this sheath can be calculated from the space charge equations used for pure electron discharges

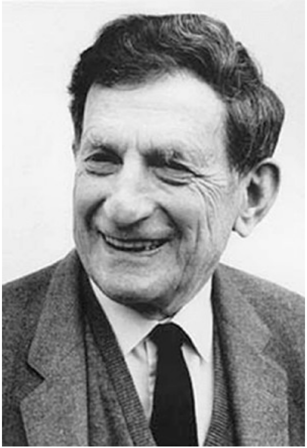
$$\frac{i}{A} = \frac{2.33 \cdot 10^{-6}}{608} \frac{V^{3/2}}{X^2} \ .$$

- Electrons are reflected from the outside surface of the sheath while all positive ions which reach the sheath are attracted to the electrode.
- The electrode is in fact perfectly screened from the discharge by the positive ion sheath.

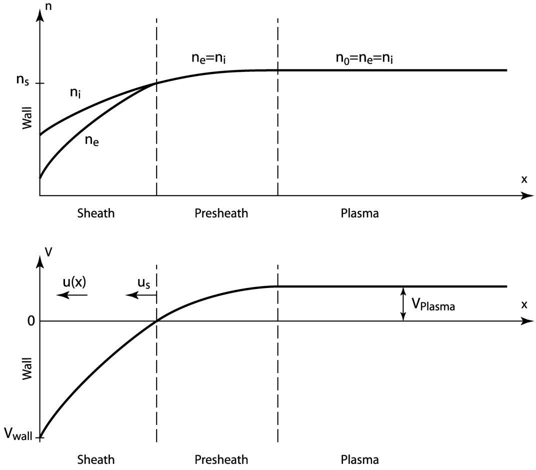
[1] Science, Volume 58, Issue 1502, pp. 290-291

David Bohm

"The ability to perceive or think differently is more important than the knowledge gained."



Bohmova teorie presheathu



David Bohm
Minimum Ionic Kinetic Energy for a Stable Sheath. New York, 1949.

Bohmovo kritérium stability vrstvy (Lieberman)

Předpoklady:

- elektrony s Maxwellovským rozdělením o teplote T_e
- teplota iontů $T_i \approx 0$
- $n_e = n_i$ na rozhraní plazmatu a sheathu

Pak zákon zachování energie pro ionty dává

$$\frac{1}{2}Mv^2(x) = \frac{1}{2}Mv_s - e\varphi(x) \tag{1}$$

Neuvažujeme-li ionizaci v sheathu

$$n_i(x)v(x) = n_{is}v_s \tag{2}$$

Předchozí rovnice lze vyřešit vzhledem k n_i

$$n_i = n_{is}\left(1 - \frac{2e(\varphi(x) - \varphi(0))}{Mv_s^2}\right)^{-\frac{1}{2}}$$

Bohmovo kritérium stability vrstvy

Elektronová hustota je dána Boltzmanovým rozdělením

$$n_e(x) = n_{es} \exp\left(\frac{(\varphi(x) - \varphi(0))}{k\,T_e}\right)$$

Průběh potenciálu lze pak obdržet z Poissonovy rovnice

$$\frac{\mathrm{d}^2(\varphi(x) - \varphi(0))}{\mathrm{d}x^2} = \frac{e}{\epsilon_0}(n_e - n_i)$$

$$= \frac{en_s}{\epsilon_0}\left[\exp\left(\frac{(\varphi(x) - \varphi(0))}{k\,T_e}\right) - \left(1 - \frac{2e\varphi}{Mv_s^2}\right)^{-\frac{1}{2}}\right]$$

Protože se jedná o kladný prostorový náboj ve vrstvě, musí být $\frac{\mathrm{d}^2\varphi}{\mathrm{d}x^2}$ záporné pro všechna $x > 0$ (a nulové pro $x = 0$)

$$\left(1 - \frac{2\,e\,(\varphi(x) - \varphi(0))}{M\,v_s^2}\right)^{-\frac{1}{2}} > \exp\left(\frac{(\varphi(x) - \varphi(0))}{k\,T_e}\right)$$

Bohmovo kritérium stability vrstvy

Po umocnění a úpravě na reciproké hodnoty dostaneme

$$\exp\left(\frac{-2\,e\,(\varphi(x)-\varphi(0))}{k\,T_e}\right) > 1 - \frac{2\,e\,(\varphi(x)-\varphi(0))}{M\,v_s^2}$$

Pokud se omezíme na malá napětí a rozvedeme levou stranu předešlé rovnice do Taylorovy řady dostaneme

$$u_s \geq \sqrt{\frac{eT_e}{M}} = u_B,$$

což je známé Bohmovo kritérium stability vrstvy.

Bohmovo kritérium stability vrstvy

To znamená, že rychlost iontů vstupujících do vrstvy musí být větší, než u_B a ukazuje nám, jak je pohyb elektronů a iontů vzákemně svázán. Cheu v roce 1974 ukázal, že fyzikální význam této podmínky je následující: Elektronová i iontová hustota klesají, ale iontová hustota klesá méně rychle, než hustota elektronů.

Jak získávají ionty tyto rychlosti? Musí existovat elektrické pole v přechodové vrstvě, aby ionty obdržely Bohmovu rychlost směrem k elektrodě. Pokud uvažujeme teplpotu iontů malou, tj. jejich rychlost neuspořádaného pohybu může být zanedbána, potom, jelikož potenciál na hranici vlastní vrstvy je V(0) vzhledem k plazmatu, lze psát

$$\frac{1}{2} m_i u(0)^2 = e\,\varphi(0)$$

$$\varphi(0) = \frac{m_i u(0)^2}{2\,e} = \frac{m_i\,k\,T_e}{2\,e\,m_i} = \frac{k\,T_e}{2\,e}$$

To neodporuje kvasineutralitě v přechodové oblasti poněvadž napětí řádu $\frac{k\,T_e}{e}$ proniká do plazmatu.

Efekt Bohmova kritéria na hodnotu rozdílu potenciálu plazmatu a plovoucího potenciálu zjištěného pomocí

Boltzmannovy statistiky je následující: Iontový tok na každý záporně nabitý předmět je zvýšen, tento rozdíl těchto

napětí se tedy zmenší.

Child-Langmuirův zákon

- Dvě nekonečné rovinné elektrody ve vzdálenosti d od sebe.
- Elektroda A emituje částice s nulovou počáteční rychlostí, $V_A = 0\,\mathrm{V}$, elektroda B absorbuje dokonale a $V_B > 0\,\mathrm{V}$.

$$v(x) = \sqrt{\frac{2eV(x)}{m}}$$

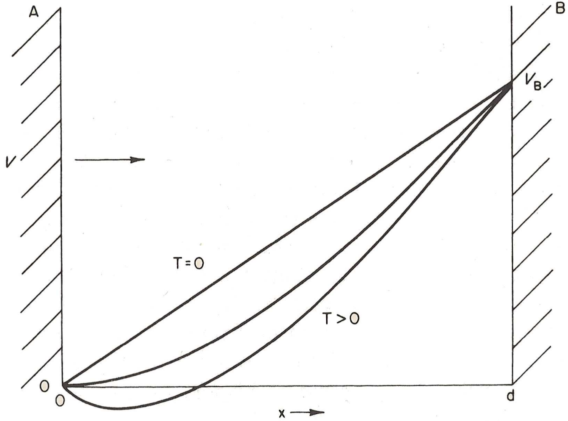
$$\rho(x) = j\sqrt{\frac{m}{2eV(x)}}$$

- Z Poissonovy rovnice lze určit průběh potenciálu mezi elektrodami

$$\frac{\mathrm{d}^2 V}{\mathrm{d}x^2} = -\frac{j}{\epsilon_0 e}\sqrt{\frac{m}{2eV(x)}} \tag{3}$$

- Předchozí rovnici (3) lze analyticky vyřešit vzhledem k j

$$|j| = \frac{4}{9}\sqrt{\frac{2e}{m}}\frac{\epsilon_0 V_0^{\frac{3}{2}}}{d^2}$$



Prostorový náboj omezuje proud. It was first used to give space-charge-limited current in vacuum diode with electrode spacing d. It can be also inverted to give the thickness of the Debye sheath as a function of the voltage drop by setting $J = j(sat)$

$$d = \frac{2}{3} (\frac{2e}{m_i})^{1/4} \frac{\varphi^{3/4}}{2\sqrt{\pi j_{sat}}}$$

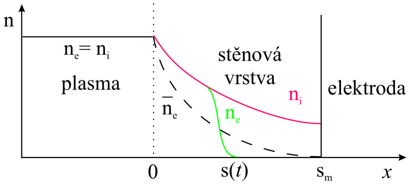
- Odpovídají-li rychlosti emitovaných částic Maxwellově distribuci

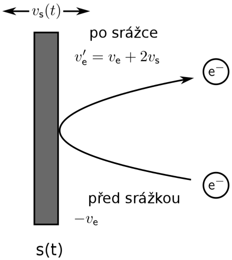
$$|j| = \frac{4}{9}\sqrt{\frac{2e}{m}}\frac{\epsilon_0(V_0 - V_{min})^{\frac{3}{2}}}{(d - d_{min})^2}\left(1 + \frac{2.66}{\sqrt{\eta}}\right)$$

- η je normalizované napětí

$$\eta = \frac{eV}{kT}$$

Stochastický ohřev v kapacitně vázaném výboji





Brian George Heil
Effects of the Dynamic Interaction between the Plasma Sheaths and Bulk on Electron Heating in Capacitively Coupled Radio-Frequency Discharges.

Thank you for your attention