

A)

$$(\sin x \cos x)' = -\sin^2 x + \cos^2 x = \cos^2 x - \sin^2 x = \cos 2x$$

$$(\sin x \cos x)' = \cos 2x$$

B)

$$\begin{aligned} \left( A^{\tan^2(2x+1)} \right)' &= A^{\tan^2(2x+1)} (\ln A) \left( 2 \tan(2x+1) \frac{1}{\cos^2(2x+1)} \cdot 2 \right) = \\ &= A^{\tan^2(2x+1)} (\ln A) \left( \frac{4 \sin(2x+1)}{\cos^2(2x+1)} \right) = A^{\tan^2(2x+1)} (\ln A) \frac{4 \sin(2x+1)}{\cos^3(2x+1)} \end{aligned}$$

$$\left( A^{\tan^2(2x+1)} \right)' = A^{\tan^2(2x+1)} (\ln A) \frac{4 \sin(2x+1)}{\cos^3(2x+1)}$$

C)

$$\left( \ln \frac{x+1}{x-1} \right)' = \frac{1}{\frac{x+1}{x-1}} \frac{(x-1) - (x+1)}{(x-1)^2} = \frac{(x-1)(-2)}{(x+1)(x-1)^2} = \frac{-2}{x^2-1}$$

$$\left( \ln \frac{x+1}{x-1} \right)' = \frac{-2}{x^2-1}$$

D)

$$\left( \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \arcsin x \right)' =$$

$$\left| \begin{array}{l} a = \frac{x}{2} \sqrt{1-x^2} \\ b = \frac{1}{2} \arcsin x \end{array} \right|$$

$$= a' + b' =$$

$$\left| \begin{array}{l} a' = \frac{1}{2} \sqrt{1-x^2} - \frac{x^2}{2\sqrt{1-x^2}} \\ b' = \frac{1}{x\sqrt{1-x^2}} \end{array} \right|$$

$$= \frac{1}{2} \sqrt{1-x^2} - \frac{x^2}{2\sqrt{1-x^2}} + \frac{1}{x\sqrt{1-x^2}} = \frac{1-x^2-x^2+1}{2\sqrt{1-x^2}} = \frac{2(1-x^2)}{2\sqrt{1-x^2}} = \sqrt{1-x^2}$$

E)

$$\left( \frac{3-x}{2} \sqrt{1-2x-x^2} + 2 \arcsin \left( \frac{1+x}{\sqrt{2}} \right) \right)' =$$

$$\begin{aligned} \text{Část1.} &= \frac{1}{2} \left( (3-x) \sqrt{-x^2-2x+1} \right)' = \frac{1}{2} \left( (-1) \sqrt{-x^2-2x+1} + \frac{1}{2} (3-x) (-x^2-2x+1)^{-\frac{1}{2}} \right) \\ &= \frac{1}{2} \left( \frac{-2(-x^2-2x+1) + (3-x)(-2x-2)}{2\sqrt{-x^2-2x+1}} \right) = \frac{1}{2} \left( \frac{4(x^2-2)}{2\sqrt{-x^2-2x+1}} \right) = \frac{x^2-2}{\sqrt{-x^2-2x+1}} \end{aligned}$$

$$\text{Část2.} = 2 \left( \arcsin \frac{1+x}{\sqrt{2}} \right)' = 2 \left( \frac{1}{\sqrt{1-\left(\frac{1+x}{\sqrt{2}}\right)^2}} \cdot \frac{1}{\sqrt{2}} \right)' = 2 \left( \frac{1}{\sqrt{\frac{2-(x^2+2x+1)}{2}}} \cdot \frac{1}{\sqrt{2}} \right)' = 2 \frac{1}{\sqrt{-x^2-2x+1}}$$

$$\text{Část1} + 2 = \frac{x^2-2}{\sqrt{-x^2-2x+1}} + \frac{2}{\sqrt{-x^2-2x+1}} = \frac{x^2}{\sqrt{-x^2-2x+1}}$$

$$\left( \frac{3-x}{2} \sqrt{1-2x-x^2} + 2 \arcsin \frac{1+x}{\sqrt{2}} \right)' = \frac{x^2}{\sqrt{-x^2-2x+1}}$$

F)

$$\left( \frac{1}{12} \ln \frac{x^4-x^2+1}{(x^2+1)^2} - \frac{1}{2\sqrt{3}} \arctan \frac{\sqrt{3}}{2x^2-1} \right)' =$$

$$= \frac{1}{12} \left( \ln \frac{x^4-x^2+1}{(x^2+1)^2} \right)' - \frac{1}{2\sqrt{3}} \left( \arctan \frac{\sqrt{3}}{2x^2-1} \right)'$$

$$\text{Část1.} = \frac{1}{12} \left( \frac{(x^2+1)^2 (4x^3-2x)(x^2+1)^2 - (x^4-x^2+1)(4x(x^2+1))}{(x^2+1)^4} \right)' =$$

$$= \frac{1}{12} \left( \frac{6(x^5-x)}{(x^4-x^2+1)(x^2+1)^2} \right) = \frac{x^5-x}{2(x^4-x^2+1)(x^2+1)^2}$$

$$\text{Část2.} = \frac{1}{2\sqrt{3}} \left( \frac{1}{1 + \left( \frac{\sqrt{3}}{2x^2-1} \right)^2} \cdot \frac{-4\sqrt{3}}{(2x^2-1)^2} \right)' = \frac{1}{2\sqrt{3}} \left( \frac{2\sqrt{3}(-2x)}{(2x^2-1)^2+3} \right)' = \frac{-2x}{(2x^2-1)^2+3}$$

$$\begin{aligned} \text{Část1} + 2 &= \frac{x^5-x}{2(x^4-x^2+1)(x^2+1)^2} - \frac{-2x}{(2x^2-1)^2+3} = \frac{x^5-x}{2(x^4-x^2+1)(x^2+1)^2} + \frac{x}{2x^4-2x^2+2} = \\ &= \frac{2x^9-2x^7+2x^5-2x^5+2x^3-2x+2x^9+2x^7+2x^3+2x}{4(x^4-x^2+1)(x^8+x^6+x^2+1)} = \frac{x^3(x^6+1)}{x^{12}+2x^6+1} = \frac{x^3(x^6+1)}{(x^6+1)^2} = \frac{x^3}{x^6+1} \end{aligned}$$

$$\left( \frac{1}{12} \ln \frac{x^4 - x^2 + 1}{(x^2 + 1)^2} - \frac{1}{2\sqrt{3}} \arctan \frac{\sqrt{3}}{2x^2 - 1} \right)' = \frac{x^3}{x^6 + 1}$$

G)

$$\begin{aligned} \left( -\sqrt{1+x^2} + x \ln(x + \sqrt{1+x^2}) \right)' &= \frac{-x}{\sqrt{1+x^2}} + \ln(x + \sqrt{1+x^2}) + x \left( \frac{1}{x + \sqrt{1+x^2}} \right) \left( 1 + \frac{x}{\sqrt{1+x^2}} \right) = \\ &= \ln(x + \sqrt{1+x^2}) + \frac{x}{x + \sqrt{1+x^2}} \frac{x + \sqrt{1+x^2}}{\sqrt{1+x^2}} - \frac{x}{\sqrt{1+x^2}} = \ln(x + \sqrt{1+x^2}) + \frac{x}{\sqrt{1+x^2}} - \frac{x}{\sqrt{1+x^2}} = \\ &= \ln(x + \sqrt{1+x^2}) \\ \left( -\sqrt{1+x^2} + x \ln(x + \sqrt{1+x^2}) \right)' &= \ln(x + \sqrt{1+x^2}) \end{aligned}$$

H)

$$\begin{aligned} \left( \frac{1}{6} \ln \frac{(x+1)^2}{x^2 - x + 1} + \frac{1}{\sqrt{3}} \arctan \frac{2x-1}{\sqrt{3}} \right)' &= \\ &= \frac{1}{6} \cdot \frac{(x^2 - x + 1)}{(x+1)^2} \cdot \frac{(2x+2)(x^2 - x + 1) - (x^2 + 2x + 1)(2x-1)}{(x^2 - x + 1)^2} + \frac{1}{\sqrt{3}} \cdot \frac{1}{1 + \left( \frac{2x-1}{\sqrt{3}} \right)^2} \cdot \frac{1}{\sqrt{3}} \cdot 2 = \\ &= \frac{(2x+2)(x^2 - x + 1) - (x^2 + 2x + 1)(2x-1)}{6(x+1)^2(x^2 - x + 1)} + \frac{2}{3 + 3 \frac{4x^2 - 4x + 1}{3}} = \\ &= \frac{2x^3 + 2 - (2x^3 + 3x^2 - 1)}{6(x+1)^2(x^2 - x + 1)} + \frac{2}{4x^2 - 4 + 4} = \frac{3x^2 - 3}{6(x+1)^2(x^2 - x + 1)} + \frac{1}{2(x^2 - x + 1)} = \\ &= \frac{(x+1)^2 - (x^2 - 1)(x^2 - x + 1)}{2(x^2 - x + 1)(x+1)^2} = \frac{x^2 + 2x + 1 - x^2 + 1}{2(x^2 - x + 1)(x+1)^2} = \frac{x+1}{(x^2 - x + 1)(x+1)^2} = \frac{1}{(x^2 - x + 1)(x+1)} = \\ &= \left[ (x^2 - x + 1)(x+1) \right]^{-1} \\ \left( \frac{1}{6} \ln \frac{(x+1)^2}{x^2 - x + 1} + \frac{1}{\sqrt{3}} \arctan \frac{2x-1}{\sqrt{3}} \right)' &= \left[ (x^2 - x + 1)(x+1) \right]^{-1} \end{aligned}$$