A)

$$y'' - 5y' + 4y = 0$$
$$\lambda^2 - 5\lambda + 4 = 0$$
$$(\lambda - 1)(\lambda - 4) = 0$$
$$\lambda_1 = 1$$
$$\lambda_2 = 4$$
$$y = C_1 \exp(4x) + C_2 \exp(x)$$

Počáteční podmínky:

$$y_{(0)} = 1$$
$$y'_{(0)} = 1$$
$$(1): C_1 + C_2 = 1$$
$$(2): 4C_1 \exp(4x) + C_2 \exp(x) = 1$$
$$4C_1 + C_2 = 1$$
$$z(1): C_1 = 1 - C_2$$
$$4(1 - C_2) + C_2 = 1$$
$$3 = 3C_2$$
$$C_2 = 1$$
$$C_1 = 1 - C_2 = 1 - 1 = 0$$

$$y = 0\exp(4x) + e^x = e^x$$

B)

$$y'' - 2y' + 2y = 4e^x(\cos x - x\sin x)$$

$$\lambda^2 - 2\lambda + 2 = 0$$

$$\lambda_{1,2} = \frac{2 \pm \sqrt{4-8}}{2} = 1 \pm i$$

$$y = C_1 \exp x(1+i) + C_2 \exp x(1-i)$$

$$P = 4e^x(\cos x - x\sin x)$$

$$\alpha = 1$$

$$\beta = 1$$

$$m = 0$$

$$n = 1$$

$$r = 1$$

Předpokládané řešení:

$$y_P = xe^x[(Ax+B)\sin x + (Cx+D)\cos x]$$

$$y_P' = e^x\left(\cos x\left(Ax^2 + Bx + 2Ax + B + Cx^2 + Dx\right) + \sin x\left(Cx^2 + Dx - Ax^2 - Bx + 2Cx + D\right)\right)$$

$$y_P'' = e^x\left[\cos x\left(4Ax + 2B + 2Cx^2 + 2Dx + 2A + 4Cx + 2D\right) + \sin x\left(-2Ax^2 - 2Bx + 4Cx + 2D + 2C + 4A - 2B\right)\right]$$

Dosazení do původní rovnice:

Pro cos*x*

$$x^2(2C - 2A - 2C + 2A) = 0x^2$$

$$0 = 0$$

$$2B + 2A + 2D - 2B = 4$$

$$A + D = 2$$

Pro sinx:

$$x^2(-2A + 2C + 2A + 2C) = 0x^2$$

$$0 = 0$$

$$x(-2B + 4C + 4A - 2D + 2B - 4C + 2D) = 4$$

$$4A = 4$$

$$A = 1$$

$$D + 2C - 2B - 2D = 0$$

$$2C - 2B = 0$$

$$B = 0$$

$$A + D = 2; A = 1 \Rightarrow D = 1$$

$$y = C_1 e^x \cos x + C_2 e^x \sin x + e^x\left(x^2 \cos x + x\sin x\right)$$

C)

$$y'' - 3y' + 2y = e^x(3 - 4x)$$
$$\lambda^2 - 3\lambda + 2 = 0$$
$$(\lambda - 2)(\lambda - 1) = 0$$
$$\lambda_1 = 1$$
$$\lambda_2 = 2$$
$$y_0 = C_1 \exp(2x) + C_2 \exp(x)$$

$$y_p = xe^x(Ax + B)$$
$$y_p = Ax^2e^x + Bxe^x$$
$$y_p' = 2Axe^x + Ax^2e^2 + Be^x + Bxe^x$$
$$y_p'' = 2Axe^x + 2Ae^x + 2Axe^x + Ax^2e^x + Be^x + Be^x + Bxe^x$$

$$Ax^2e^x + 4Axe^x + 2Ae^x + 2Be^x + Bxe^x - 6Axe^x - 3Ax^2e^x + 2Ax^2e^x + 2Bxe^x - 3Be^x - 3Bxe^x = e^x(3 - 4x)$$

Vykrátíme : e^x

$$(1 - 3 + 2)Ax^2 + (4 - 6)Ax + 2A + (2 - 3)B + Bx(1 + 2 - 3) = 3 - 4x$$
$$-2xA + 2A - B = 3 - 4x$$
$$2Ax = 4x \Rightarrow A = 2$$

$$2A - B = 3 \Rightarrow B = 1$$

$$y = y_0 + y_p$$
$$y = C_1 \exp(2x) + C_2 \exp(x) + xe^x(2x + 1)$$