13:

a:

$$\frac{\partial^3 t}{\partial x^2 \partial y}$$

$$t(x,y) = xy\ln y + x^y$$

$$\frac{\partial t}{\partial x} = y\ln y + yx^{y-1}$$

$$\frac{\partial^2 t}{\partial t^2} = y(y-1)x^{y-2}$$

$$\frac{\partial^3 t}{\partial y \partial t^2} = \frac{1}{x^2}\left(2yx^y + y^2x^y\ln x - \left(x^y + yx^{yl}\ln x\right)\right) = x^{y-2}\left(2y + y^2\ln x - y\ln x - 1\right)$$

b:

$$\frac{\partial^4 t}{\partial x \partial y \partial z^2}$$

$$t(x,y,z) = xy^2z^3 + x^2\sin^2(xz) + yz + x + y + z$$

$$\frac{\partial t}{\partial y} = 2xz^3y + z + 1$$

$$\frac{\partial^2 t}{\partial x \partial y} = 2yz^3$$

$$\frac{\partial^3 t}{\partial x \partial y \partial z} = 6yz^2$$

$$\frac{\partial^4 t}{\partial x \partial y \partial z^2} = 12yz$$

14:

$$V = -\aleph \frac{mM}{r}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\overrightarrow{Fg} = --\aleph \frac{mM}{r^2}\frac{\vec{r}}{|r|} = -\aleph \frac{mM}{r^3}\vec{r}$$

$$V = -\aleph \frac{mM}{\sqrt{x^2 + y^2 + z^2}} = -\aleph\, mM\left(x^2 + y^2 + z^2\right)^{-\frac{1}{2}}$$

$$\frac{\partial V}{\partial x} = -\left(-\frac{1}{2}\right)2x\aleph\, mM\left(x^2 + y^2 + z^2\right)^{-\frac{3}{2}} = \aleph\, mM\frac{x}{r^3}$$

$$\frac{\partial V}{\partial y} = -\left(-\frac{1}{2}\right)2y\aleph\, mM\left(x^2 + y^2 + z^2\right)^{-\frac{3}{2}} = \aleph\, mM\frac{y}{r^3}$$

$$\frac{\partial V}{\partial z} = -\left(-\frac{1}{2}\right)2z\aleph\, mM\left(x^2 + y^2 + z^2\right)^{-\frac{3}{2}} = \aleph\, mM\frac{z}{r^3}$$

$$gradV = \left(\aleph\, mM\frac{x}{r^3};\aleph\, mM\frac{y}{r^3};\aleph\, mM\frac{z}{r^3}\right) = \aleph\, mM\frac{1}{r^3}(x,y,z) = \aleph\, mM\frac{1}{r^3}\vec{r} = -\overrightarrow{Fg}$$

15:
funkce t
vektor **A**

$\vec{A} = (Ax; Ay, Az)$

$tA = (tAx; tAy; tAz)$

$$rot(tA) = grad(t) \times A + trotA$$

Levá strana:

$$rot(\vec{tA}) = \left(\frac{\partial(tAz)}{\partial y} - \frac{\partial(tAy)}{\partial z}; \frac{\partial(tAx)}{\partial z} - \frac{\partial(tAz)}{\partial x}; \frac{\partial(tAy)}{\partial x} - \frac{\partial(tAx)}{\partial y} \right) =$$

$$= \left(\frac{\partial(t)}{\partial y} Az + t\frac{\partial Az}{\partial y} - \frac{\partial(t)}{\partial z} Ay + t\frac{\partial Ay}{\partial z}; \frac{\partial(t)}{\partial z} Ax + t\frac{\partial Ax}{\partial z} - \frac{\partial(t)}{\partial x} Az + t\frac{\partial Az}{\partial x}; \frac{\partial(t)}{\partial x} Ay + t\frac{\partial Ay}{\partial x} - \frac{\partial(t)}{\partial y} Ax + t\frac{\partial Ax}{\partial y} \right)$$

Pravá strana:

$$gradt = \left(\frac{\partial t}{\partial x}; \frac{\partial t}{\partial y}; \frac{\partial t}{\partial z} \right)$$

$$gradt \times \vec{A} = \left(Az\frac{\partial t}{\partial x} - Ay\frac{\partial t}{\partial z}; Ax\frac{\partial t}{\partial z} - Az\frac{\partial t}{\partial x}; Ay\frac{\partial t}{\partial x} - Ax\frac{\partial t}{\partial y} \right)$$

$$\vec{trotA} = \left(t\frac{\partial Az}{\partial y} + t\frac{\partial Ay}{\partial z}; t\frac{\partial Ax}{\partial z} + t\frac{\partial Az}{\partial x}; t\frac{\partial Ay}{\partial x} + t\frac{\partial Ax}{\partial y} \right)$$

$$grad(t) \times A + trotA =$$

$$= \left(\frac{\partial(t)}{\partial y} Az + t\frac{\partial Az}{\partial y} - \frac{\partial(t)}{\partial z} Ay + t\frac{\partial Ay}{\partial z}; \frac{\partial(t)}{\partial z} Ax + t\frac{\partial Ax}{\partial z} - \frac{\partial(t)}{\partial x} Az + t\frac{\partial Az}{\partial x}; \frac{\partial(t)}{\partial x} Ay + t\frac{\partial Ay}{\partial x} - \frac{\partial(t)}{\partial y} Ax + t\frac{\partial Ax}{\partial y} \right) = rot(\vec{tA})$$

$L = P$