F2422 - HW10 - 17. května 2006

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Obsah

1 Zadání

Určete koeficienty a_k, b_k ve Fourierově řadě pro následující funkci $f_{(x)}$ (zakreslete rovněž její graf!): $f_{(x)}$ má periodu 2π, $f_{(x)} = x^2$ pro $-\pi \leq x < \pi$.

2 Rozbor

$f_{(x)} = x^2; x \in (-\pi; \pi) \ldots T_0 = 2\pi$
Trigonometrickou Fourierovou řadou funkce f nazýváme řadu:

$$f_{(x)} = \frac{a_0}{2} + \sum_{k=1}^{\infty} [a_k \cos(k\omega_0 x) + b_k \sin(k\omega_0 x)]$$

$f_{(x)} = x^2; x \in (-\pi; \pi) \ldots T_0 = 2\pi$

$a_k = \frac{2}{T_0} \int_{-\pi}^{\pi} f(x) \cos(k\omega_0 x) dx$

$b_k = \frac{2}{T_0} \int_{-\pi}^{\pi} f(x) \sin(k\omega_0 x) dx$

$\omega_0 = \frac{2\pi}{T_0}$

$a_k = \frac{2}{2\pi} \int_{-\pi}^{\pi} x^2 \cos(kx) dx$

$b_k = \frac{2}{2\pi} \int_{-\pi}^{\pi} x^2 \sin(kx) dx$

3 Výpočet integrálů

3.1 Integrál $\int_{-\pi}^{\pi} f(x) \cos(k\omega_0 x) dx$

$$A = \int_{-\pi}^{\pi} x^2 \cos(kx) dx$$

$$A = \left| \begin{array}{cc} u' = \cos(kx) & u = \frac{1}{k}\sin(kx) \\ v = x^2 & v' = 2x \end{array} \right|_{-\pi}^{\pi}$$

$$A = \left[\frac{x^2}{k} \sin(kx) \right]_{-\pi}^{\pi} - \frac{2}{k} \int_{-\pi}^{\pi} x \sin(kx) dx$$

$$A = \frac{2\pi^2 \sin(k\pi)}{k} - \frac{2}{k} \left| \begin{array}{cc} u' = sin(kx) & u = -\frac{1}{k}\cos(kx) \\ v = x & v' = 1 \end{array} \right|_{-\pi}^{\pi}$$

$$A = \frac{2\pi^2\sin(k\pi)}{k} + \left[\frac{2x}{k^2}\cos(kx)\right]_{-\pi}^{\pi} - \frac{2}{k^2}\int_{-\pi}^{\pi}\cos(kx)dx$$
$$A = \frac{2\pi^2\sin(k\pi)}{k} - \frac{4\pi\cos(k\pi)}{k^2} + \frac{2\sin(k\pi)}{k^3}$$
$$A = \frac{k^2\pi^2\sin(k\pi) - 2\sin(k\pi) + 2k\pi\cos(k\pi)}{k^3}$$

3.2 Integrál $\int_{-\pi}^{\pi} f(x)\sin(k\omega_0 x)dx$

$$B = \int_{-pi}^{\pi} x^2\sin(kx)dx$$
$$B = \left| \begin{array}{cc} u' = sin(kx) & u = -\frac{1}{k}\cos(kx) \\ v = x^2 & v' = 2x \end{array} \right|_{-\pi}^{\pi}$$
$$B = \left[-\frac{x^2}{k}\cos(kx)\right]_{-\pi}^{\pi} + \frac{2}{k}\int_{-\pi}^{\pi} x\cos(kx)dx$$
$$B = 0 + \frac{2}{k}\left| \begin{array}{cc} u' = cos(kx) & u = \frac{1}{k}\sin(kx) \\ v = x & v' = 1 \end{array} \right|_{-\pi}^{\pi}$$
$$B = \left[\frac{2x}{k}\sin(kx)\right]_{-\pi}^{\pi} - \frac{2}{k^2}\int_{-\pi}^{\pi}\sin(kx)dx$$
$$B = 0 - 0$$
$$B = 0$$

4 Výpočet koeficientů a_k a b_k

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\cos(kx)dx$$
$$a_k = \frac{1}{\pi}\cdot 2\frac{k^2\pi^2\sin(k\pi) - 2\sin(k\pi) + 2k\pi\cos(k\pi)}{k^3}$$
$$a_k = 2\frac{k^2\pi^2\sin(k\pi) - 2\sin(k\pi) + 2k\pi\cos(k\pi)}{\pi k^3}$$

Protože $k \in Z$ tak členy se $\sin(kx)$ resp. $\sin(k\pi)$ vypadnou, zbyde tedy jen:

$$a_k = \frac{4k\pi\cos(k\pi)}{\pi k^3}$$
$$a_k = \frac{4\cos(k\pi)}{k^2}$$
$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\sin(kx)dx = \frac{1}{\pi}\cdot 0 = 0$$

5 Členy a_k

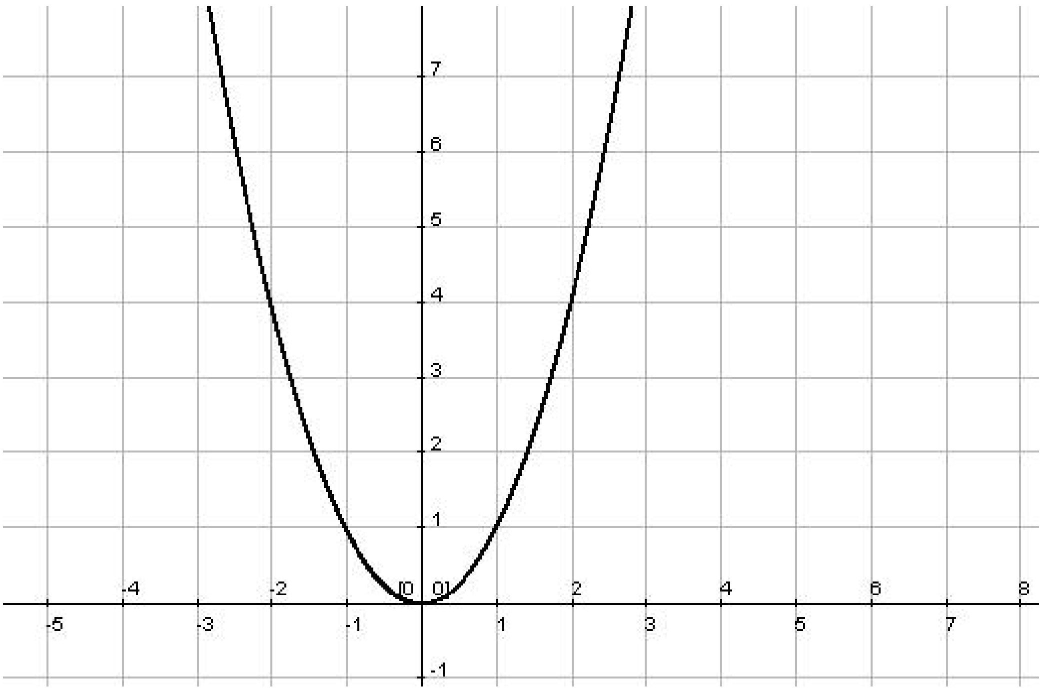
Z vyšších řádků plyne:
$a_1 = -4$
$a_2 = \frac{4}{2^2}$
$a_3 = \frac{-4}{3^2}$
$a_4 = \frac{4}{4^2}$

6 Fourierova řada

$$f_{(x)} = \frac{a_0}{2} - 4\cos(x) + \frac{4}{2^2}\cos(2x) - \frac{4}{3^2}\cos(3x) + \frac{4}{4^2}\cos 4x...$$

$$f_{(x)} = \frac{a_0}{2} - 4\left[\cos(x) - \frac{1}{2^2}\cos(2x) + \frac{1}{3^2}\cos(3x) - \frac{1}{4^2}\cos 4x + ...\right]$$

7 Obrázek



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