

F4120 — Teoretická mechanika

Závorky

Zadání

Vypočtete Poissonovy závorky složek L_x , L_y vektoru momentu hybnosti a dále Poissonovu závorku $\{L^2, L_z\}$, kde $L^2 = L_x^2 + L_y^2 + L_z^2$.

$$\mathbf{L} = (yp_z - zp_y, zp_x - xp_z, xp_y - yp_x)$$

$$\{L_x, L_y\} = \frac{\partial L_x}{\partial p_z} \frac{\partial L_y}{\partial x} - \frac{\partial L_y}{\partial p_z} \frac{\partial L_x}{\partial x} + \frac{\partial L_x}{\partial p_y} \frac{\partial L_y}{\partial y} - \frac{\partial L_y}{\partial p_y} \frac{\partial L_x}{\partial y} + \frac{\partial L_x}{\partial p_z} \frac{\partial L_y}{\partial z} - \frac{\partial L_y}{\partial p_z} \frac{\partial L_x}{\partial z}$$

$$\{L_x, L_y\} = yp_x - xp_y = -L_z$$

$$\{L^2, L_z\} = \{L_x^2, L_z\} + \{L_y^2, L_z\} + \{L_z^2, L_z\}$$

$$\{L^2, L_z\} = 2L_x \{L_x, L_z\} + 2L_y \{L_y, L_z\} + 2L_z \{L_z, L_z\}$$

Víme, že: $\{L_x, L_z\} = -L_y$, $\{L_y, L_z\} = L_x$ a $\{L_z, L_z\} = 0$

$$\{L^2, L_z\} = -2L_x L_y + 2L_y L_x$$

$$\{L^2, L_z\} = 0$$