# Refractometer for tracking changes in the refractive index of air near 780 nm

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A new system, consisting of a double-channel Fabry–Perot etalon and laser diodes emitting around 780 nm, is described and proposed for use for measuring air-refractive index. The principle of this refractometer is based on frequency measurements between optical laser sources. It permits quasi-instantaneous measurement with a resolution of better than $10^{-9}$ and uncertainty in the $10^{-8}$ range. Some preliminary results on the stability of this system and the measurements of the refractive index of air with this apparatus are presented. The first measurements of the index of air at 780 nm are, within an experimental uncertainty of the order of $2 \times 10^{-8}$, in agreement with the predicted values by the so-called revised Edlén equations. This result is, to the best of our knowledge, the first to extend to the near IR the validity of the revised Edlén equation derived for the wavelength range of 350–650 nm. © 1998 Optical Society of America

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## 1. Introduction

One of the most important sources of error limiting the accuracy of length measurements by interferometric techniques arises from the uncertainty and the fluctuations of the refractive index of air. Most such interferometric techniques use, as reference sources, lasers that are frequency stabilized to atomic or molecular lines. Despite the high degree of stability (a relative uncertainty below $10^{-11}$) of such reference lasers, because of the definition of the Mètre,[1] the associated wavelength $\lambda_\nu$ is well defined only in vacuum. Practical measurements of length or distance, on the other hand, are performed mostly in an air environment where refractive-index fluctuations result in variations in the wavelength ($\lambda_a = \lambda_\nu/n$) of the reference laser source. Generally the refractometers used for measuring air refractive index are based on the classical technique of counting interference fringes and interferometric phase measurements. In such systems the optical path of a cell of length $\ell$, placed in the interferometer, is changed by the quantity $2\ell(n - 1)$ when the air is evacuated from the cell. This variation is related to wavelength $\lambda_\nu$ of the source that illuminates the interferometer by $2\ell(n - 1) = (k + \varepsilon)\lambda_\nu$, where $k$ is an integer and $\varepsilon$ is a fractional number. With this method the index of air can be measured with a level of uncertainty fixed essentially by those of $\ell$ and $\varepsilon$. (Generally the uncertainty of $\lambda_\nu$ can be neglected.)

Until now, the best refractometers used for measuring air refractive index reach relative uncertainties limited to the $10^{-7}$ level.[2] Furthermore their response times are rather slow compared with the rate of variation of the index of air associated with changes in climatic conditions (temperature, pressure, humidity, and $CO_2$ concentration). In the case of our refractometer the index of air is determined only from beat-frequency measurements between optical laser sources. The uncertainties in the measurements can reach the $10^{-8}$ level. This level allows one to improve the accuracy of length-measurement techniques based on interferometric methods in air.

As shown in Section 2, the most important application of this apparatus concerns the development of a new type of laser reference where the wavelength is stabilized, even if $n$ varies, as opposed to the frequency as in classical optical frequency standards.[1] This type of reference is insensitive to fluctuations in

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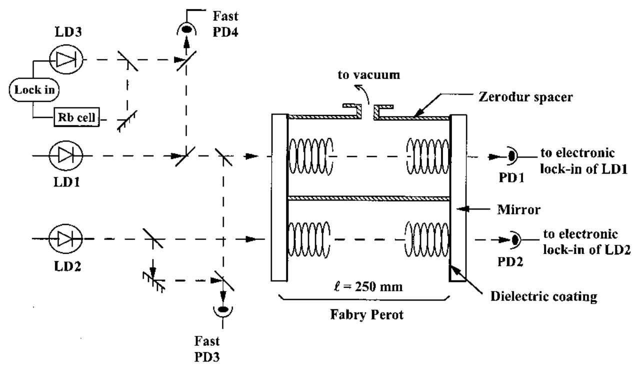


Fig. 1. Experimental setup of our refractometer.

air refractive index and exhibits a long-term stability $\delta\lambda/\lambda$ of $10^{-8}$.

## 2. Principle of the Measurement

Our apparatus is shown in Fig. 1. The central element of the air refractometer is a dual-channel plane–plane Fabry–Perot interferometer with a spacer of length $\ell = 250$ mm, made from Zerodur (thermal coefficient expansion, $10^{-8}\,°\mathrm{C}^{-1}$). This cavity is described in more detail in Section 3. Each channel is illuminated independently by a single-mode laser diode operating around 780 nm. One channel can be evacuated when the other is kept at atmospheric pressure. The beat frequency between laser diodes LD1 and LD2 is measured with photodetector PD3 and that between LD1 and LD3 by PD4. Photodiodes PD1 and PD2 are used to lock lasers LD1 and LD2 to transmission peaks of their respective cavities. When the frequency of a laser is locked to a transmission peak of this Fabry–Perot etalon, it follows that $\ell = k(\lambda/2)$, where $k$ is an integer and $\lambda$ is the wavelength of the laser.

The measurement method is as follows: First, air is present in both channels of the interferometer. With the help of a beat-frequency measurement, with photodetector PD3, it is straightforward to ensure that each laser is locked to the same peak $k$ of frequency $\nu_k = k[c/(2n\ell)]$ (where $c$ is the speed of light in vacuum and $n$ is the refractive index of air). In this situation, $\Delta\nu = \nu_1 - \nu_2$ is in principle equal to zero, and since wavelengths $\lambda_{1a}$ and $\lambda_{2a}$ (both in air) are equal to $\lambda_{1a} = \lambda_{2a} = 2\ell/k$, the corresponding frequencies are given by

$$\nu_1 = \nu_2 = \frac{c}{n\lambda_{1a}} = \frac{c}{n\lambda_{2a}}. \quad (1)$$

When the pressure of the first channel, illuminated by laser LD1, is slowly reduced, laser LD1 remains locked to the same peak $k$. After evacuation of this channel (the residual pressure is ~1 Pa), wavelength $\lambda_{1\nu}$ in vacuum remains equal to

$$\lambda_{1\nu} = 2\ell/k = \lambda_{2a}. \quad (2)$$

This consistency means that the interference pattern between the two mirrors of the Fabry–Perot cavities remains unchanged during the pumping procedure. On the other hand, the frequency of the laser LD1 becomes

$$\nu_1{}^* = c/\lambda_{1\nu}, \quad (3)$$

giving rise to a beat frequency $\Delta\nu = \nu_1{}^* - \nu_1$ between the two diode lasers equal to $(n - 1)\nu_1$, which is of the order of 105 GHz. The new frequency $\nu_1{}^*$ of laser LD1 can be calibrated by comparison with a reference frequency $\nu_{ref}$, in our case laser LD3 locked to a hyperfine component of the rubidium $D_2$ line. Thus

$$\nu_1 = \nu_1{}^* - \Delta\nu, \qquad \nu_1{}^* = \nu_{ref} + \Delta\nu^*. \quad (4)$$

Finally, from our knowledge of $\nu_{ref}$ and the almost instantaneous measurements of $\Delta\nu$ and $\Delta\nu^*$, we have an instantaneous determination of the refractive index of air. Explicitly

$$n = \frac{\nu_1{}^*}{\nu_1} = \frac{\nu_{ref} + \Delta\nu^*}{\nu_{ref} + \Delta\nu^* - \Delta\nu}. \quad (5)$$

From Eq. (5) one can see that with this technique measurement of the index of air does not depend on the value of $k$ or on $\ell$, which is generally not the case with the other traditional interferometric methods.

There are two main difficulties inherent in our method. The first difficulty is scanning continuously the frequency of laser LD1 across ~105 GHz while the laser remains locked to the same transmission peak of the Fabry–Perot without mode hopping. The second difficulty is measuring the associated beat frequency $\Delta\nu$, which lies close to this value. To address the first problem we employ distributed Bragg reflector (DBR) laser diodes (Yokogawa Company). As well as providing single-mode operation with a narrow spectral linewidth (1 MHz), these diodes can be tuned continuously over a wide wavelength range (>150 GHz). There are two possible ways to overcome the second difficulty, namely, measurement of the large frequency difference:

The first way is a direct measurement of $\Delta\nu$ and uses three-wave mixing on a Schottky diode.[3] Here the second harmonic of a stabilized microwave field of frequency $\Omega_{rf}$ (approximately equal to 52 GHz) is mixed with optical laser beams of frequencies $\nu_1$ and $\nu_1{}^*$. This mixing gives a beat signal at frequency $[(\nu_1{}^* - \nu_1) - 2\,\Omega_{rf}]$, which can be measured directly.

The other method uses the fact that beat frequency $\Delta\nu$ can be compared with the free spectral range ($\mathrm{FSR} = c/2n\ell$) of the Fabry–Perot. In this case we measure the frequency difference between $\nu_1{}^*$ and $\nu_1 + p[c/(2n\ell)]$. Integer $p$ is such that this difference is less than 1 FSR. More details about this procedure are in Section 5.

In practice, $\nu_{ref}$ is known to within a few kilohertz and $\Delta\nu$ and $\Delta\nu^*$ can be measured with an accuracy well below 1 MHz, which means that we can determine $n$ with a relative uncertainty of better than $10^{-8}$ (one standard deviation). An interesting characteristic of this refractometer is the fact that the value of wavelength $\lambda_{2a}$ of laser LD2 is insensitive to fluctu-

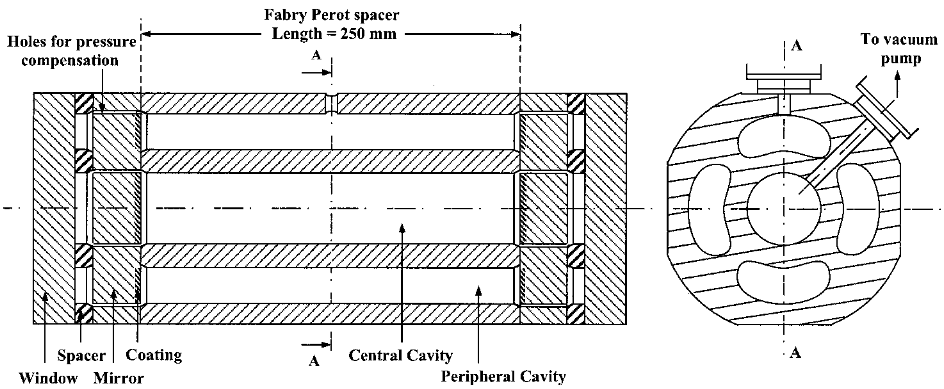


Fig. 2. Design (longitudinal and transverse sections) of the Fabry–Perot etalon.

ations in air refractive index. In fact, from Eqs. (2), (3), and (4), the wavelength of laser LD2 is given by $\lambda_{2a} = c/(\nu_{ref} + \Delta\nu^*)$.

This property is used currently in our laboratory to provide an optical wavelength reference.

## 3. Construction and Design of the Refractometer

To minimize the deformations of the mirrors when making the vacuum inside the channel 1, we make the double Fabry–Perot etalon from a single piece of Zerodur on which two fused silica 20-mm-thick mirrors, with appropriate coatings, are fixed by optical contact (Fig. 2). Behind each mirror lies a 5-mm-thick silica spacer (of the same geometry as the Zerodur spacer). One side of this spacer is fixed to the mirror by optical contact. The other side supports a 20-mm-thick optically flat silica window also fixed by optical contact. As one can see in Fig. 2, there are holes in the mirrors to equalize the pressure on each side of the mirror during the pumping procedure. With this geometry, when the central cavity of the Fabry–Perot is evacuated, the small deformations undergone by the windows will not affect the flatness of the mirrors.

## 4. Preliminary Tests

### A. Frequency Stability Measurements

To characterize the stability of the interferometer, we measure the fluctuations $\delta\nu_i$ of the frequency $\nu_k = k[c/(2\ell)]$ of the Fabry–Perot by comparing $\nu_k$ with $\nu_{ref}$ with a heterodyne method (Fig. 3). The reference frequency $\nu_{ref}$ is obtained with saturated absorption spectroscopy of atomic rubidium (Rb) in a vapor cell. We employ the crossover resonance between the hyperfine transitions ($5S_{1/2}$, $F_g = 2 \rightarrow 5P_{3/2}$, $F_e = 2$) and ($5S_{1/2}$, $F_g = 2 \rightarrow 5P_{3/2}$, $F_e = 3$) of the $^{87}$Rb–$D_2$ line. The frequency of this resonance is given by[4] $\nu_{ref} = 384{,}227{,}981.877$ MHz.

The beat frequency $\delta\nu_i$ between this reference and laser diode LD1 locked to the transmission peak of frequency $\nu_k$ of one cavity of the Fabry–Perot is measured during an integration time $\tau$ of 1 s. $N$ equivalent measurements made at equal time intervals $\tau$ allow one to perform a statistical analysis of the frequency fluctuations $\delta\nu_i$ by the relative Allan variance[5]:

$$\sigma = \frac{1}{\nu_k}\left[\sum_{i=1}^{N}\frac{(\delta\nu_i - \delta\nu_{i-1})^2}{2(N-1)}\right]^{1/2},$$

where $\delta\nu_i = \nu_k(t_i) - \nu_{ref}$ with $t_i = i\tau$, and $i = 1, 2, \ldots, N$. The results in Fig. 4 show the good short-term stability of the Fabry–Perot ($\approx 5 \times 10^{-11}$ in 100 s), which corresponds to residual frequency fluctuations. Figure 4 also shows the stability of the reference frequency, measured by the same technique using a second system ($\approx 4 \times 10^{-12}$ in 100 s).

The long-term stability of the interferometer is limited by the temperature changes of the Zerodur.

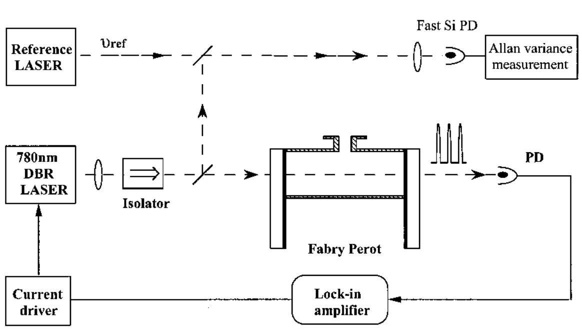


Fig. 3. Block diagram for stability measurements of the Fabry–Perot.

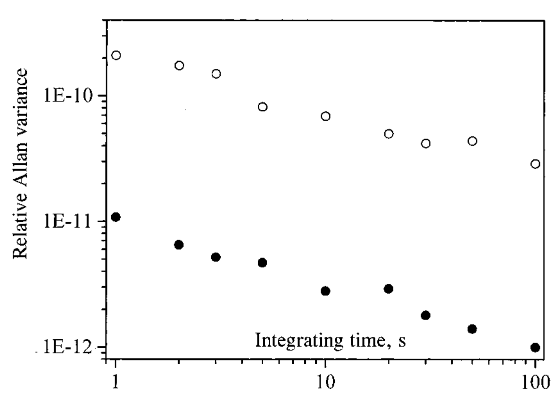


Fig. 4. Relative standard deviation plot against integration time: ●, stability of the laser diode locked to the $D_2$ line of rubidum; ○, stability of the laser diode locked to a transmission peak of the Fabry–Perot etalon.

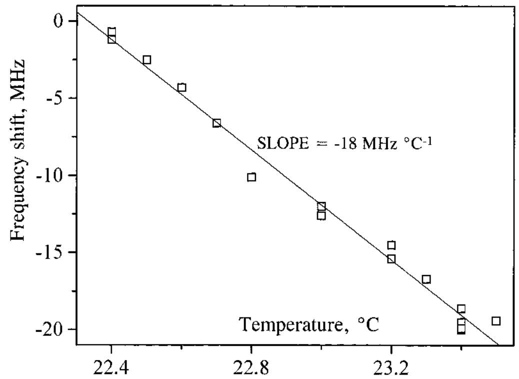


Fig. 5. Change in the beat frequency between the reference frequency and the laser diode locked to the transmission peak of one cavity of the Fabry–Perot versus temperature.

The linear thermal expansion coefficient of this material has been evaluated by measuring, over a long period of time, the shift in the beat note between our reference and a laser diode locked on the peak of frequency $\nu_k$ of the Fabry–Perot. Figure 5 shows a decrease of $\nu_k$ by $\sim$18 MHz °C$^{-1}$, which corresponds to a relative increase in the length of the interferometer by $\sim 5 \times 10^{-8}$ °C$^{-1}$. This value agrees with the longitudinal thermal expansion coefficient in the range of 0–50 °C of class I Zerodur glass ceramics ($\pm 5 \times 10^{-8}$ °C$^{-1}$) specified by the Schott Company. In actual fact, since thermal fluctuations lead to almost identical changes in the lengths of both cavities, their influence on our measurements can be neglected.

### B. Effect of Parallelism of Mirror Faces and Air Pressure

To measure the refractive index of air with relative uncertainty in the range of $10^{-8}$, we require that the length difference between the two cavities of the Fabry–Perot (central and peripheral cavities) remain the same to within $\lambda/300$. This difference corresponds to a shift in the beat note between the corresponding peak $k$ of the two cavities smaller than 4 MHz. Such a shift can be measured easily. Any variation in the length of the cavities due to a pressure differential can be measured by the following procedure.

Initially the two cavities are in air and we choose a peak number $k$ of the peripheral cavity so that the corresponding frequency $\nu_2$ is close to frequency $\nu_{\text{ref}}$. The quantity $\Delta\nu = \nu_2 - \nu_{ref}$ is measured. When the central cavity is evacuated, the external pressure leads to a reduction in the length, giving rise to a frequency shift of peak $k$:

$$\nu_2{}^i = \nu_2 + c_2.$$

The term $c_2$ is of the order of 30 MHz, corresponding to a change in the cavity length by $\approx\lambda/40$. To first order, the corresponding deformation is assumed to be equivalent in both cavities. Since our method of measurement is independent of the length of the Fabry–Perot cavities, linear changes in this length will not affect the accuracy of the measurements. However, placing an upper limit on the residual nonlinearity of the deformation between the two cavities of 10% introduces a differential frequency shift of the order of 3 MHz ($\lambda/400$) that corresponds to a relative uncertainty of the order of $8 \times 10^{-9}$ in the measurement of $n$. This effect is in fact the principal limitation on the accuracy of our apparatus.

### C. Sensitivity of the Apparatus to Misalignment Errors

Any small change $\delta\theta$ in the inclination of the laser beam with respect to the optic axis of the Fabry–Perot interferometer induces corresponding modifications of the path difference. Such a change would introduce, first, deformation in the shape of the transmission peak (easily seen when the interferometer is illuminated with a periodic laser frequency ramp) and, second, variation in the beat frequency between the reference frequency and the laser locked to the Fabry–Perot. A relative uncertainty of the order of $10^{-8}$ needs to be angularly adjusted to within $10^{-4}$ rad. This adjustment implies a beat-frequency measurement uncertainty of less than 4 MHz. This requirement is easily satisfied by observing the position of the beat-frequency signal on the spectrum analyzer. Consequently we can neglect errors due to misalignments.

## 5. Determination of the Refractive Index of Air: First Measurements

Since the Schottky diode and the rf mixer system are not yet operational, the beat frequency ($\Delta\nu \approx 105$ GHz) is measured indirectly. Our method involves a similar concept that is used to achieve absolute distance measurement.[6] The results for the refractive index of air are compared with those derived from the revised Edlén equation[7] used for ambient atmospheric conditions over the wavelength range of 350–650 nm.

### A. Measurement Procedure

First, we start with both cavities of the Fabry–Perot at atmospheric pressure. Frequency $\nu_k$ of the transmission peak, on which diode lasers LD1 (for the central cavity) and LD2 (for the peripheral cavity) are locked, is, respectively, $\nu_1$ and $\nu_2 = \nu_1 + c_{12}$. Correction term $c_{12}$ is of the order of 70 MHz, corresponding to the shift between peak $k$ of the two cavities. When the central cavity of the interferometer is slowly evacuated, frequency $\nu_1$ changes and goes to $\nu_1{}^*$. We obtain this last frequency by measuring the beat frequency between $\nu_1{}^*$ and $\nu_r$. For more clarity in the description of the measurement procedure, the different frequencies used here are indicated in Fig. 6 ($\nu_r$ represent $\nu_{\text{ref}}$).

Frequency $\nu_1$ is chosen so that the difference between $\nu_1{}^*$ and $\nu_r$ corresponds to a few hundred megahertz in order that it can be measured by conventional beat-frequency techniques:

$$\nu_1{}^* = \nu_r + \delta\nu_{1r}. \tag{6}$$

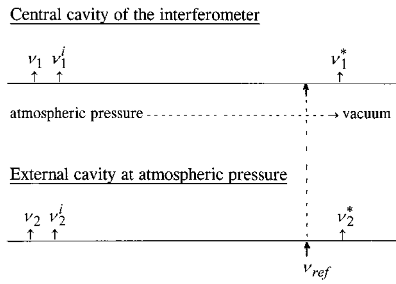


Fig. 6. Representation of the procedure used for measuring the refractive index of air.

The refractive index of air is given by the ratio $\nu_1{}^*/\nu_1{}^i$, where $\nu_1{}^i$ is the frequency $\nu_1$ shifted by $c_1$ because the central cavity is under vacuum:

$$\nu_1{}^i = \nu_1 + c_1. \tag{7}$$

During evacuation of the central cavity of the interferometer the peripheral channel is maintained at atmospheric pressure but undergoes a similar compression effect, giving a frequency $\nu_2{}^i$ related to $\nu_2$ by a correction term $c_2$:

$$\nu_2{}^i = \nu_2 + c_2. \tag{8}$$

Next, laser LD2 is no longer locked to the $k$th peak of frequency $\nu_2$ but instead is scanned manually, and, after we have counted an integer number $p$ of fringes, it is relocked to the peak number $(k + p)$ so that the frequency of this laser is varied from $\nu_2{}^i$ to $\nu_2{}^*$:

$$\nu_2{}^* = \nu_2{}^i + p\,\frac{c}{2n\ell}. \tag{9}$$

Frequency $\nu_2{}^*$ lies close to $\nu_1{}^*$ to within less than one FSR of the Fabry–Perot, i.e.,

$$\Delta\nu_{\mathrm{FP}}{}^{\mathrm{air}} = \frac{c}{2n\ell}.$$

The calibration of $\nu_2{}^*$ in terms of $\nu_r$ gives

$$\nu_2{}^* = \nu_r + \delta\nu_{2r}, \tag{10}$$

where $\delta\nu_2$ represents the beat frequency between the reference source and diode laser LD2 at frequency $\nu_2{}^*$. From Eqs. (6)–(10) we obtain

$$\nu_1{}^i = \nu_2{}^i - c_2 - c_{12} + c_1, \qquad \nu_2{}^i = \nu_r + \delta\nu_{2r} - p\,\frac{c}{2n\ell}. \tag{11}$$

Finally, from Eqs. (6) and (11) the refractive index of air can be expressed in the form

$$n = \frac{\nu_r + \delta\nu_{1r} + p\,\dfrac{c}{2\ell}}{\nu_r + \delta\nu_{2r} + c_1 - c_2 - c_{12}}. \tag{12}$$

To first order we suppose that corrections $c_1$ and $c_2$ are the same, which means that the atmospheric pressure causes the lengths of both cavities to change by the same amount. With this procedure, we can take measurements of the refractive index of air and also track its fluctuations at any time. This tracking is achieved thanks to the quasi-instantaneous measurement of the two beat frequencies $\delta\nu_{1r}$ and $\delta\nu_{2r}$.

With this method, the uncertainty in the index of air is less than $2 \times 10^{-8}$. This limitation is mainly due to uncertainty in the correction $(c_1 - c_2)$. In our case, all the corrections have been evaluated to within 3 MHz (one standard deviation), whereas the FSR in vacuum was measured in our laboratory by a beat-frequency technique to be $599.594 \pm 0.005$ MHz.

Figure 7 shows the results of the measurement carried out continuously for a few hours on different days. The calculated values are obtained from the measured values of temperature, atmospheric pressure, and air humidity by the revised Edlén equations[7]:

$$(n-1)_{tp} = \frac{(p/Pa)(n-1)_s}{96095.43} \times \frac{[1 + 10^{-8}\,(0.601 - 0.00972t/°C)p/Pa]}{(1 + 0.0036610\,t/°C)},$$

where $(n-1)_{tp}$ is the refractive index of air at temperature $t$ and pressure $p$ and $(n-1)_s$ is given by the dispersion equation

$$(n-1)_s \times 10^8 = 8342.54 + 2406147[130 - (\lambda^{-1}/\mu\mathrm{m}^{-1})^2]^{-1} + 15998[38.9 - (\lambda^{-1}/\mu\mathrm{m}^{-1})^2]^{-1},$$

where $\lambda^{-1} = 1/\lambda$ represents the wave number expressed in $\mu\mathrm{m}^{-1}$.

For moist air containing a partial pressure $f$ of water vapor the refractive index is

$$n_{tpf} = n_{tp} - (f/Pa)[3.7345 - 0.0401(\lambda^{-1}/\mu\mathrm{m}^{-1})^2] \times 10^{-10}.$$

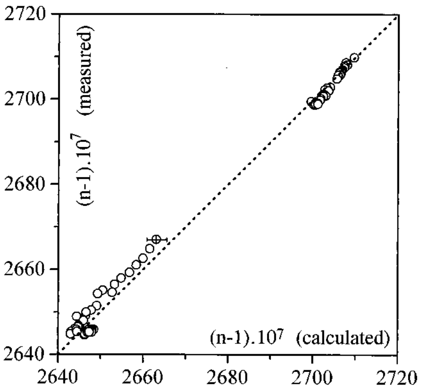


Fig. 7. Plot of measured values of the refractive index of air and those calculated by the revised Edlèn equation. The error bars are derived from the uncertainties quoted in the various optical measurements and instruments. The dotted line represents the bisecting line.

The uncertainty in the calculated values, $\sim 2 \times 10^{-7}$, is associated with the resolution of the instruments used for determining air temperature, pressure, and humidity. The absolute values of the refractive index measured by the beat-frequency technique are in the same range as those calculated above, but more accurate.

## 6. Conclusion

We have developed a novel refractometer based on a two-channel Fabry–Perot interferometer illuminated by tunable frequency laser diodes. Measurements of air refractive index are made simply by a beat-frequency technique that provides a resolution of better than $10^{-9}$. The first results with this apparatus are in agreement with those calculated with the empirical Edlén formulas commonly used for calculation of the index of air, but the relative uncertainty of our measurements reaches the level $2 \times 10^{-8}$. Note that the revised Edlén formulas derived for the wavelength range of 350–650 nm are also valid near 780 nm to within $10^{-7}$.

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