Scalable Effective Models for Superconducting Nanostructures

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Abstract

As superconducting (SC) hybrid nanostructures grow increasingly complex, with multiple quantum dots, multiterminal superconductors, and devices operating far from equilibrium, the need for scalable and accurate theoretical modeling becomes ever more pressing. Capturing essential physical mechanisms while keeping simulations tractable poses a major challenge, especially when superconducting correlations and strong interactions coexist.

In this talk, I will present a versatile framework for constructing effective models of superconducting heterostructures described by the generalized superconducting Anderson Impurity Model (SC-AIM) with multiple impurities and leads. Our method, Chain Expansion (ChE) [1], maps superconducting leads onto finite one-dimensional chains, enabling efficient simulations both in and out of equilibrium while retaining essential physical properties. The mapping can be tailored either to optimally capture low-energy physics or to reproduce the full-bandwidth tunneling self-energy.

Furthermore, I will show that by exploiting the recently introduced geometrical factor [2], we can reduce any number of leads coupled to a single quantum dot to an effective single-chain representation. This significantly simplifies the treatment of multiterminal devices without loss of accuracy.

We derived simple analytical expressions that enable rapid construction of chains optimized for different computational tasks, including ground-state calculations and real-time evolution. Benchmarking against Numerical Renormalization Group (NRG) results demonstrates that ChE-based models agree remarkably well with full SC-AIM solutions across a broad parameter space. High accuracy is achieved even for short chains accessible to Exact Diagonalization (ED), while longer chains remain tractable thanks to the one-dimensional structure compatible with Density Matrix Renormalization Group (DMRG) methods. Time-dependent calculations using ChE also show excellent agreement with non-equilibrium Green's function approaches.

After validating ChE on benchmark systems, we apply it to complex device architectures involving serial and parallel quantum-dot configurations. I will discuss their tunability, rich phase diagrams, and the role of parity in shaping Andreev bound states (ABS). [1] Scalable Effective Models for Superconducting Nanostructures: Applications to Double, Triple, and Quadruple Quantum Dots, D. Bobok, L. Frk, V. Pokorný, M. Žonda, arXiv:2508.18465 (accepted in PRB) [2] Hidden symmetry in interacting-quantum-dot-based multiterminal Josephson junctions, P. Zalom, M. Žonda, T. Novotný, Phys. Rev. Lett. 132 (12), 126505 (2024)