On Neural Network Based Automated Theorem Prover For Minimal Logic

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Outline

Minimal Logic

		Provable?		
Formula	Name	Classical	Intuitionistic	Minimal
$\neg A \supset (A \supset B)$	**The law of contradiction**	yes	yes	no
$A \vee \neg A$	**The principle of excluded middle**	yes	no	no

Outline

Problems of Gentzen style sequent system G1

- Generation of proofs which are permutations of each other.

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- Choice of the rule to be applied.

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System GM

$$\frac{A, \Gamma \Rightarrow B}{\Gamma \Rightarrow A \supset B}\,(\supset_R) \qquad \frac{\Gamma \Rightarrow A \quad \Gamma \xrightarrow{B} C}{\Gamma \xrightarrow{A \supset B} C}\,(\supset_L)$$

$$\frac{\Gamma \xrightarrow{A} C}{\Gamma \xrightarrow{A \& B} C}\,(\&_{L1}) \qquad \frac{\Gamma \xrightarrow{B} C}{\Gamma \xrightarrow{A \& B} C}\,(\&_{L2})$$

$$\frac{\Gamma \Rightarrow A \quad \Gamma \Rightarrow B}{\Gamma \Rightarrow A \& B}\,(\&_R) \qquad \frac{A, \Gamma \Rightarrow C \quad B, \Gamma \Rightarrow C}{\Gamma \xrightarrow{A \vee B} C}\,(\vee_L)$$

$$\frac{\Gamma \Rightarrow A}{\Gamma \Rightarrow A \vee B}\,(\vee_{R1}) \qquad \frac{\Gamma \Rightarrow B}{\Gamma \Rightarrow A \vee B}\,(\vee_{R2})$$

$$\frac{A, \Gamma \xrightarrow{A} B}{A, \Gamma \Rightarrow B}\,(C) \qquad \frac{}{\Gamma \xrightarrow{A} A}\,(ax) \qquad \frac{\Gamma \Rightarrow A}{\Gamma \xrightarrow{\perp} A}\,(\perp)$$

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- Choice of the rule to be applied. (Partially solved)

Example of loop in system GM

$$p \& p \supset p \Rightarrow p$$

Example of loop in system GM

$$\frac{(p\&p) \supset p \xrightarrow{(p\&p)\supset p} p}{p\&p \supset p \Rightarrow p}\ (C)$$

Example of loop in system *GM*

$$\dfrac{\dfrac{(p\&p)\supset p \xrightarrow{p} p \qquad\qquad (p\&p)\supset p \Rightarrow p\&p}{(p\&p)\supset p \xrightarrow{(p\&p)\supset p} p}\,(\supset_L)}{p\&p\supset p\Rightarrow p}\,(C)$$

Example of loop in system GM

$$
\cfrac{\cfrac{\cfrac{}{(p\&p)\supset p \xrightarrow{\;p\;} p}\,(ax) \qquad \cfrac{(p\&p)\supset p \Rightarrow p \qquad (p\&p)\supset p \Rightarrow p}{(p\&p)\supset p \Rightarrow p\&p}\,(\&_R)}{(p\&p)\supset p \xrightarrow{(p\&p)\supset p} p}\,(\supset_L)}{p\&p\supset p \Rightarrow p}\,(C)
$$

$$
\dfrac{
\dfrac{}{(p\&p)\supset p \xrightarrow{\;p\;} p}\,(ax)
\qquad
\dfrac{
\dfrac{\cdots}{(p\&p)\supset p \Rightarrow p}
\qquad
\dfrac{\cdots}{(p\&p)\supset p \Rightarrow p}
}{(p\&p)\supset p \Rightarrow p\&p}\,(\&_R)
}{
\dfrac{(p\&p)\supset p \xrightarrow{(p\&p)\supset p} p}{p\&p\supset p \Rightarrow p}\,(C)
}\,(\supset_L)
$$

Outline

Systems GM^{Hist} with history mechanisms

We introduce two systems for propositional fragment of minimal logic:

- GM^{Hist} with Swiss history

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- GM^{Hist} with Swiss history
- GM^{Hist} with Scottish history

GM^{Hist} with Swiss history

$$\frac{A,\Gamma \Rightarrow B;\, \varepsilon}{\Gamma \Rightarrow A \supset B;\, H}\ (\supset R_1),\ \textit{if } A \notin \Gamma \qquad \frac{\Gamma \Rightarrow B;\, H}{\Gamma \Rightarrow A \supset B;\, H}\ (\supset R_2),\ \textit{if } A \in \Gamma$$

$$\frac{A,\Gamma \Rightarrow \bot;\, \varepsilon}{\Gamma \Rightarrow \neg A;\, H}\ (\neg R_1),\ \textit{if } A \notin \Gamma \qquad \frac{\Gamma \Rightarrow \bot;\, H}{\Gamma \Rightarrow \neg A;\, H}\ (\neg R_2),\ \textit{if } A \in \Gamma$$

$$\frac{\Gamma \Rightarrow A;\, (C,H) \qquad \Gamma \xrightarrow{B} C; H}{\Gamma \xrightarrow{A \supset B} C;\, H}\ (\supset L),\ \textit{if } C \notin H$$

$$\frac{\Gamma \Rightarrow A;\, (C,H) \qquad \Gamma \xrightarrow{\bot} C; H}{\Gamma \xrightarrow{\neg A} C;\, H}\ (\neg L),\ \textit{if } C \notin H$$

$$\frac{\Gamma \xrightarrow{A} C;\, H}{\Gamma \xrightarrow{A \wedge B} C;\, H}\ (\wedge L_1) \qquad \frac{\Gamma \xrightarrow{B} C;\, H}{\Gamma \xrightarrow{A \wedge B} C;\, H}\ (\wedge L_2)$$

$$\frac{\Gamma \Rightarrow A;\, H \qquad \Gamma \Rightarrow B;\, H}{\Gamma \Rightarrow A \wedge B;\, H}\ (\wedge R)$$

$$\frac{A,\Gamma \Rightarrow C;\, \varepsilon \qquad B,\Gamma \Rightarrow C;\, \varepsilon}{\Gamma \xrightarrow{A \vee B} C;\, H}\ (\vee L),\ \textit{if } A, B \notin \Gamma$$

$$\frac{\Gamma \Rightarrow A;\, H}{\Gamma \Rightarrow A \vee B;\, H}\ (\vee R_1) \qquad \frac{\Gamma \Rightarrow B;\, H}{\Gamma \Rightarrow A \vee B;\, H}\ (\vee R_2)$$

$$\frac{A,\Gamma \xrightarrow{A} B;\, H}{A,\Gamma \Rightarrow B;\, H}\ (C)^{*} \qquad \frac{\Gamma \Rightarrow A;\, (A,H)}{\Gamma \xrightarrow{\bot} A;\, H}\ (\bot) \qquad \frac{}{\Gamma \xrightarrow{A} A;\, H}\ (ax)$$

GM^{Hist} with Scottish history

$$\frac{A,\Gamma \Rightarrow B;\ \{B\}}{\Gamma \Rightarrow A \supset B;\ H}\ (\supset R_1),\ \textit{if } A \notin \Gamma \qquad \frac{A,\Gamma \Rightarrow \bot;\ \{\bot\}}{\Gamma \Rightarrow \neg A;\ H}\ (\neg R_1),\ \textit{if } A \notin \Gamma$$

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$$\frac{\Gamma \Rightarrow A;\ (A,H) \qquad \Gamma \Rightarrow B;\ (B,H)}{\Gamma \Rightarrow A \wedge B;\ H}\ (\wedge R),\ \textit{if } A,B \notin H$$

$$\frac{A,\Gamma \Rightarrow C;\{C\} \quad B,\Gamma \Rightarrow C;\ \{C\}}{\Gamma \xrightarrow{A \vee B} C;\ H}\ (\vee L),\ \textit{if } A,B \notin \Gamma$$

$$\frac{\Gamma \Rightarrow A;\ (A,H)}{\Gamma \Rightarrow A \vee B;\ H}\ (\vee R_1),\ \textit{if } A \notin H \qquad \frac{\Gamma \Rightarrow B;\ (B,H)}{\Gamma \Rightarrow A \vee B;\ H}\ (\vee R_2),\ \textit{if } B \notin H$$

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Comparisions between history mechanisms

GM^{Hist} with Swiss history

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Comparisions between history mechanisms

GM^{Hist} with Swiss history

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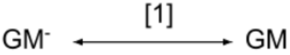
- requires less checkings.
- is slower.
- makes some unnecessary steps.

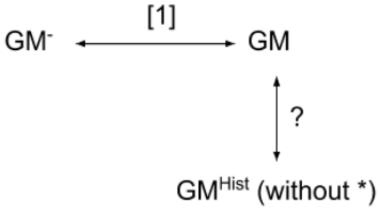
Equivalence between GM and GM^{Hist}



Theorem 1

The systems GM^- [1] and GM are equivalent. That is, a sequent S is provable in GM^- if and only if S is provable in GM. [3]

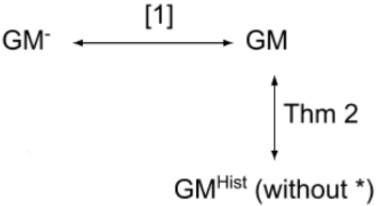
Equivalence between GM and GM^{Hist}



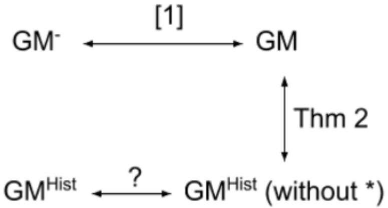
Equivalence between GM and GM^{Hist}

Theorem 2

*The systems GM and GM^{Hist} (without *) are equivalent. That is, a sequent S is provable in GM if and only if $S; \epsilon$ (the sequent with empty history) is provable in GM^{Hist} (without *).*

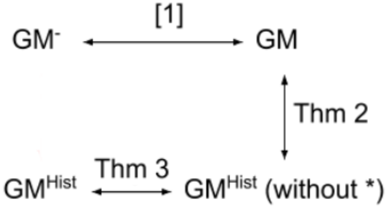


Equivalence between GM and GM^{Hist}

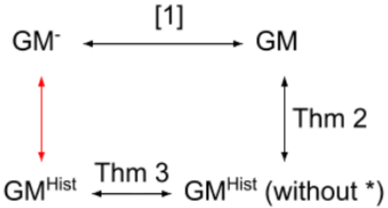


Theorem 3

*The calculus GM^{Hist} with condition * placed on rule (C) is equivalent to GM^{Hist} without the extra condition.*



Equivalence between GM and GM^{Hist}



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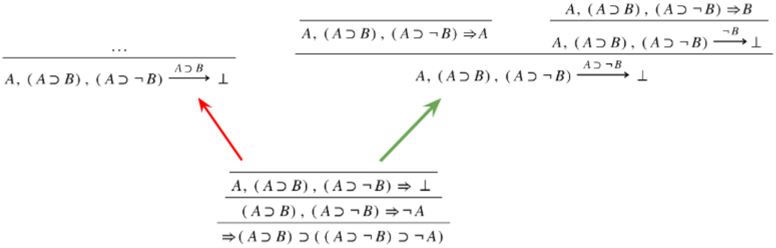
Example of rule selection problem

$$\dfrac{\dfrac{\overline{A, (A \supset B), (A \supset \neg B) \Rightarrow \bot}}{(A \supset B), (A \supset \neg B) \Rightarrow \neg A}}{\Rightarrow (A \supset B) \supset ((A \supset \neg B) \supset \neg A)}$$

Example of rule selection problem



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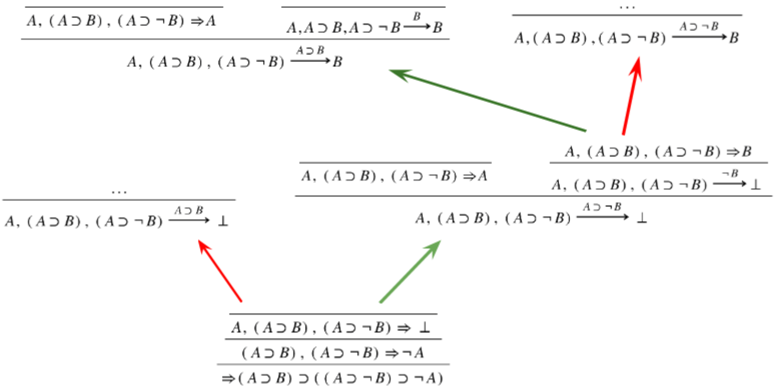

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- An automated theorem prover *SwProv* based on GM^{Hist} system with Swiss history is developed.

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- As a result more than 50000 training data points were generated. Data was divided into train/validation/test parts with 80%-10%-10% proportions.

Formula representation via autoencoders

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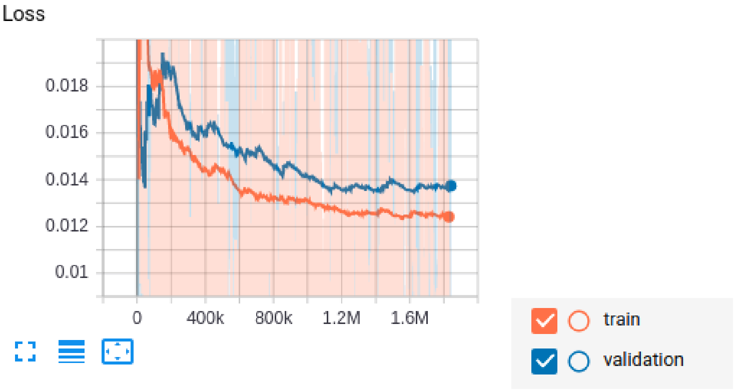
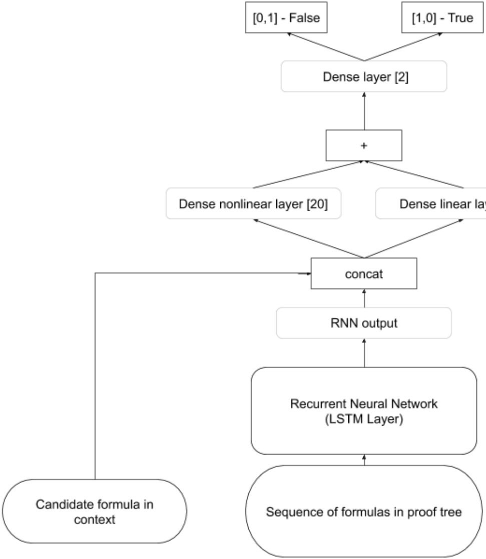
- As a final step autoencoder is trained to get fixed length encoding for each sequent.

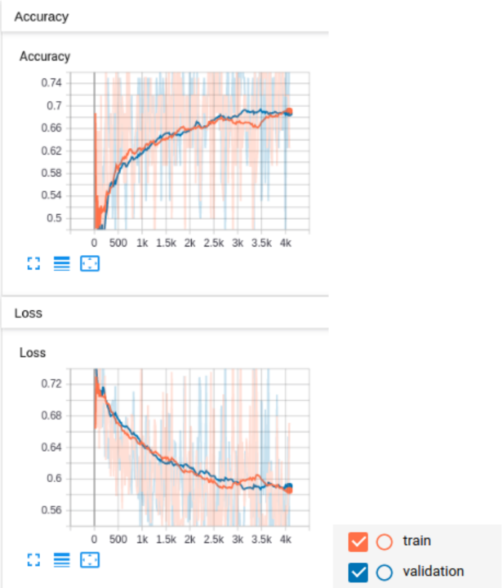
Autoencoder training process



Recurrent Neural Network



RNN training process



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Some optimizations for neural network

- Inference reduction.

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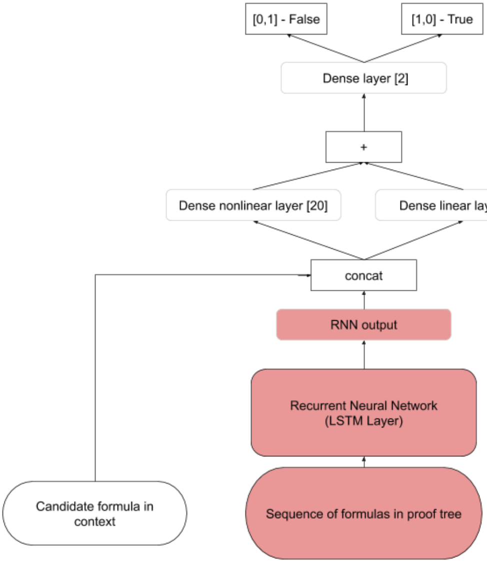
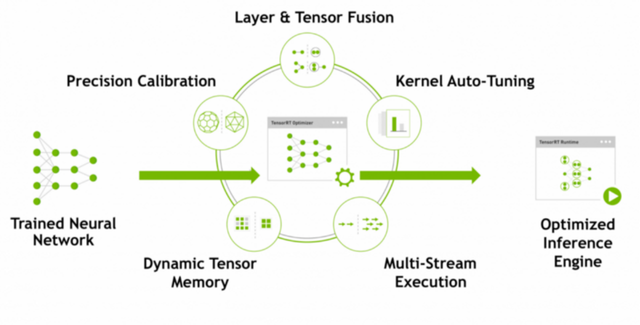
- Inference on different GPUs with various architecures.

Inference reduction



NVIDIA TensorRT optimization



Inference on different GPUs with various architecures

GPU	Architecture	Without TensorRT (inf/sec)	TensorRT float32 (inf/sec)	TensorRT float16 (inf/sec)
Nvidia P100	Pascal	870	1200	1400
Nvidia K80	Kepler	250	350	400
Nvidia V100	Volta	1100	1600	3100
Nvidia T4	Turing	740	1000	1900
Nvidia Jetson Nano	Maxwell	50	65	70

Outline

Results

Formula	SwProv (10^{-3} sec)	SwNNProv (10^{-3} sec)
$(A \supset B) \supset (A \supset C) \supset (A \supset (B \supset C))$	2	30
$((\neg\neg A \supset A) \supset A) \bigvee (\neg A \supset \neg A) \bigvee (\neg\neg A \supset \neg\neg A) \bigvee (\neg\neg A \supset A)$	3	9
$((C \supset ((A \supset B) \supset ((A \supset \neg B) \supset \neg A))) \supset ((((A \supset B) \supset ((A \supset \neg B) \supset \neg A)) \supset D) \supset (C \supset D)))$	35	20
$((((P \supset Q) \supset ((Q \supset R) \supset (P \supset R)))) \supset R) \supset ((Q \supset R) \supset (((((P \supset Q) \supset ((Q \supset R) \supset (P \supset R)))) \bigvee Q) \supset R)))$	37	16
$(((A \supset B) \supset ((A \supset \neg B) \supset \neg A)) \supset B) \supset ((B \supset C) \supset (((A \supset B) \supset ((A \supset \neg B) \supset \neg A)) \supset C))$	65	29
$((((P \supset Q) \supset ((Q \supset R) \supset (P \supset R)))) \supset B) \supset ((((P \supset Q) \supset ((Q \supset R) \supset (P \supset R)))) \supset C) \supset ((((P \supset Q) \supset ((Q \supset R) \supset (P \supset R)))) \supset (B \,\&\, C))$	82	65
$(((((P \supset R) \supset ((Q \supset R) \supset ((P \bigvee Q) \supset R)))) \supset Q) \supset (((((P \supset R) \supset ((Q \supset R) \supset ((P \bigvee Q) \supset R)))) \supset \neg Q) \supset \neg(((P \supset R) \supset ((Q \supset R) \supset ((P \bigvee Q) \supset R))))))$	129	15
$(((G \supset A) \supset J) \supset ((P \bigvee (Q \,\&P)) \supset P) \supset E) \supset (((H \supset B) \supset I) \supset C \supset J \supset (A \supset H) \supset F \supset G \supset (((C \supset C) \supset I) \supset ((P \bigvee (Q \,\&P)) \supset P)) \supset (A \supset C) \supset (((F \supset A) \supset B) \supset I) \supset E)$	869	174
$(((((G \supset A) \supset J) \supset D \supset E) \supset (((H \supset B) \supset I) \supset C \supset J \supset (A \supset H) \supset F \supset G \supset (((C \supset B) \supset I) \supset D) \supset (A \supset C) \supset (((F \supset A) \supset B) \supset I) \supset E)) \,\&\, B) \supset ((((G \supset A) \supset J) \supset D \supset E) \supset (((H \supset B) \supset I) \supset C \supset J \supset (A \supset H) \supset F \supset G \supset (((C \supset B) \supset I) \supset D) \supset (A \supset C) \supset (((F \supset A) \supset B) \supset I) \supset E))$	1359	96

References

[1] Bolibekyan H. R., Chubaryan A. A., On some proof systems for I.Johansson's minimal logic of predicates, Proceedings of the Logic Colloquium, 2003, p. 56.
[2] Baghdasaryan A., Bolibekyan H., On Some Systems of Minimal Propositional Logic with History Mechanism, Proceedings of the Logic Colloquium, 2017, p. 80.
[3] Bolibekyan H. R., Baghdasaryan A. R., On Some Systems of Propositional Minimal Logic with Loop Detection, Reports of National Academy of Sciences of Armenia, vol. 119 (2019), no. 2, pp. 110–115
[4] Howe, J.M., Theorem Proving and Partial Proof Search for Intuitionistic Propositional Logic Using a Permutation-free Calculus with Loop Checking. University of St Andrews Research Report CS/96/12, 1996.

Thanks for your attention!