The design and modeling of microstructured optical fiber

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Outline

- What are these fibers (and why should I care)?
- The guiding mechanisms: index-guiding and band gaps
- Finding the guided modes
- Small corrections (with big impacts)

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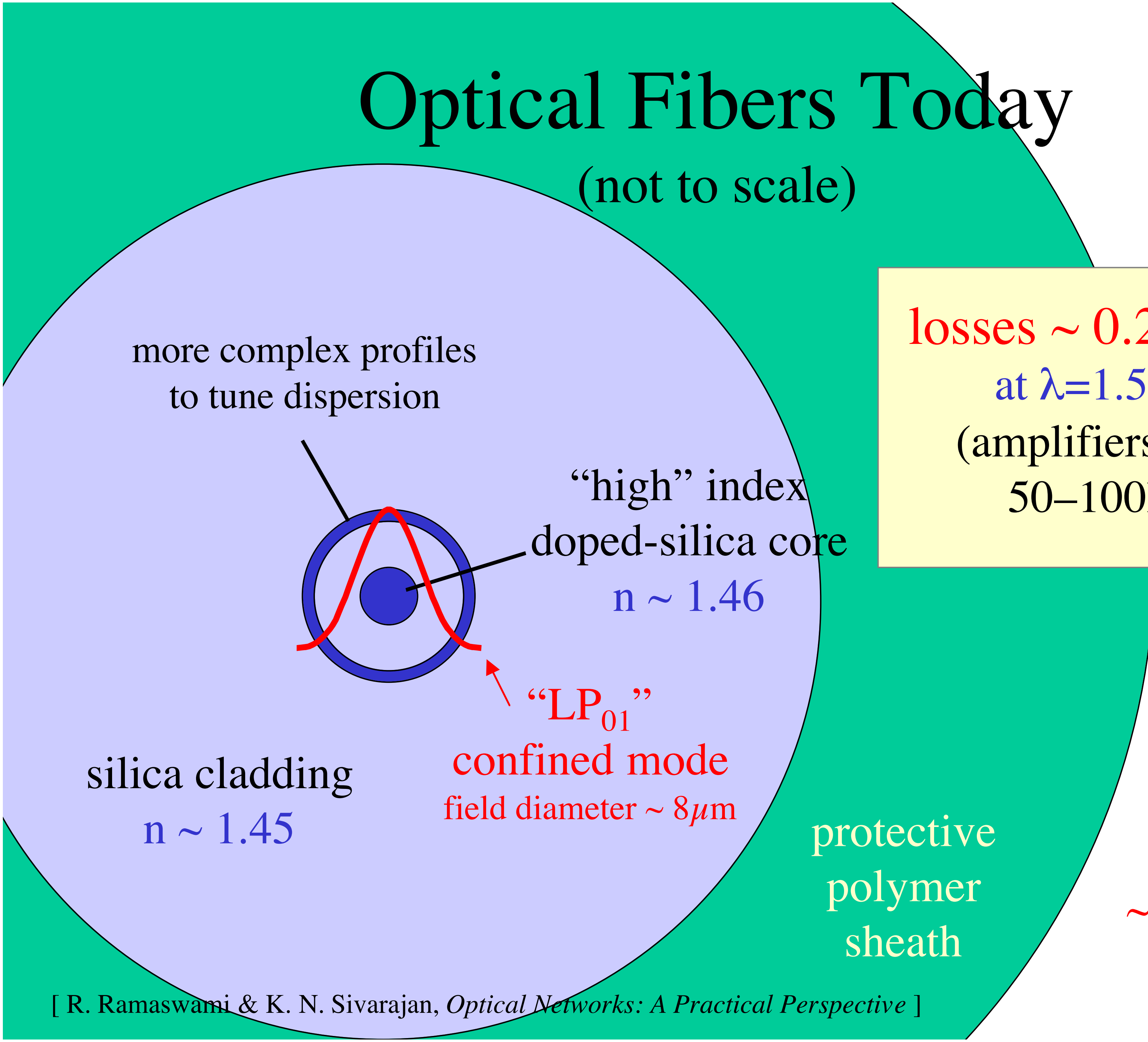
Optical Fibers Today

(not to scale)



losses ~ 0.2 dB/km
at λ=1.55μm
(amplifiers every
50–100km)

but this is
~ as good as
it gets…

[R. Ramaswami & K. N. Sivarajan, *Optical Networks: A Practical Perspective*]

The Glass Ceiling: *Limits of Silica*

Loss: amplifiers every 50–100km

…limited by Rayleigh scattering (molecular entropy)

…cannot use "exotic" wavelengths like 10.6μm

Nonlinearities: after ~100km, cause dispersion, crosstalk, power limits

(limited by mode area ~ single-mode, bending loss)

also cannot be made (very) **large** for compact nonlinear devices

Radical modifications to dispersion, polarization effects?

…tunability is limited by low index contrast

High Bit-Rates

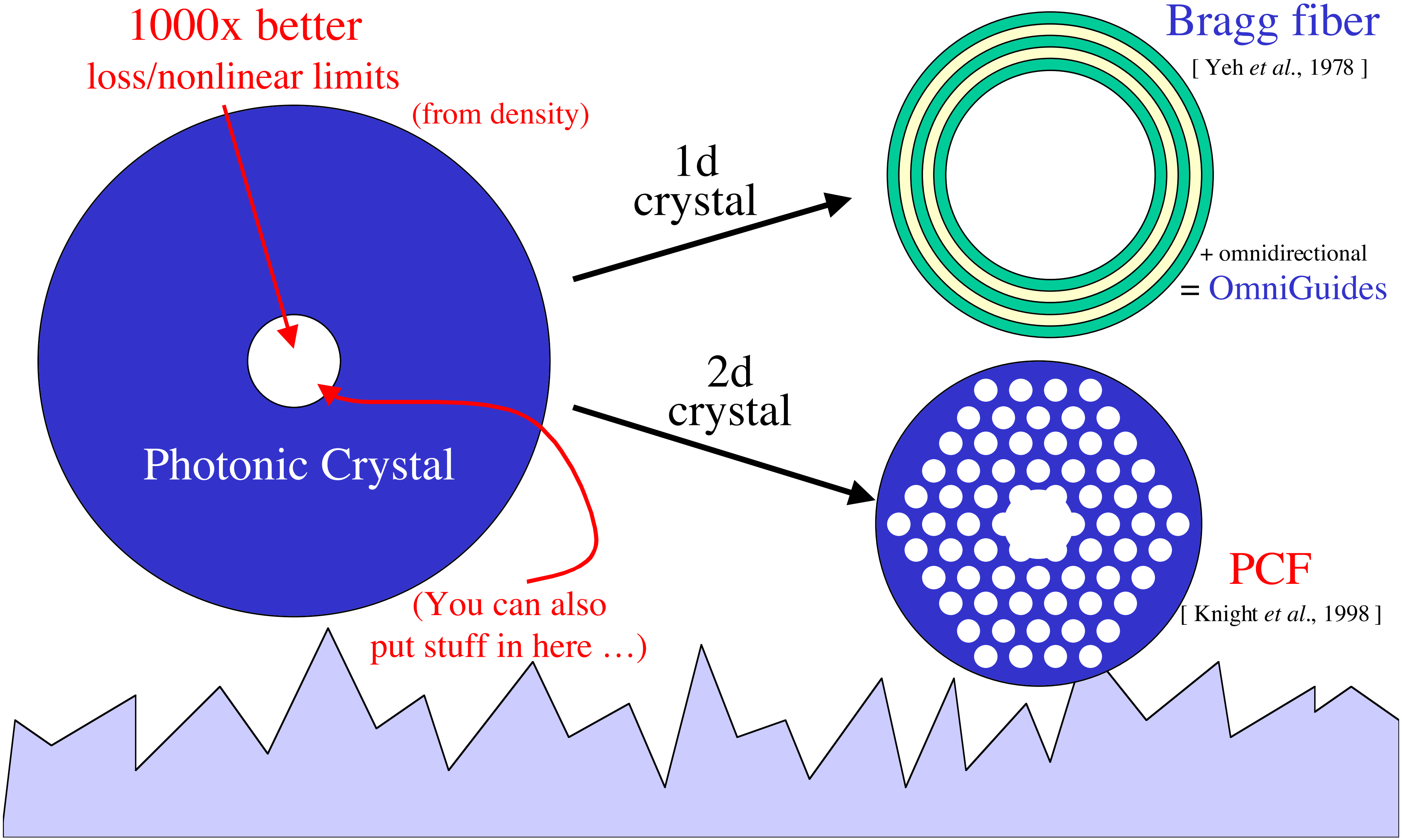
Compact Devices

Long Distances

Dense Wavelength Multiplexing (DWDM)

Breaking the Glass Ceiling:

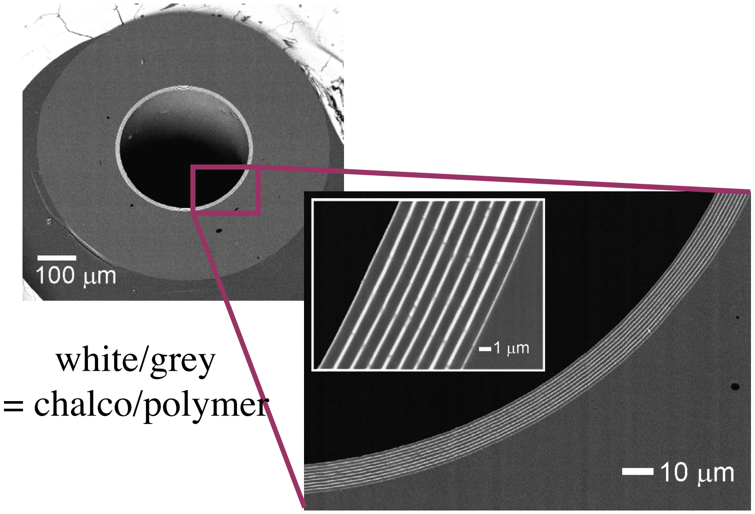
Hollow-core Bandgap Fibers



Breaking the Glass Ceiling:

Hollow-core Bandgap Fibers

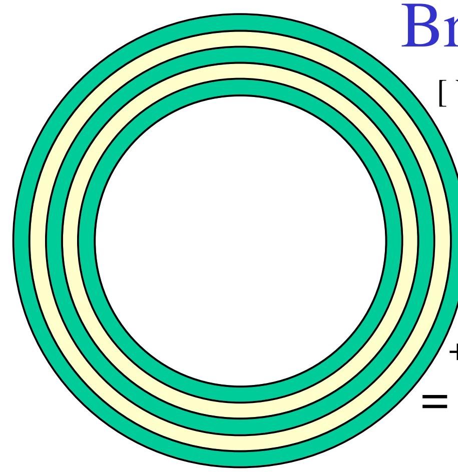
[figs courtesy
Y. Fink *et al.*, MIT]



white/grey
= chalco/polymer

Bragg fiber

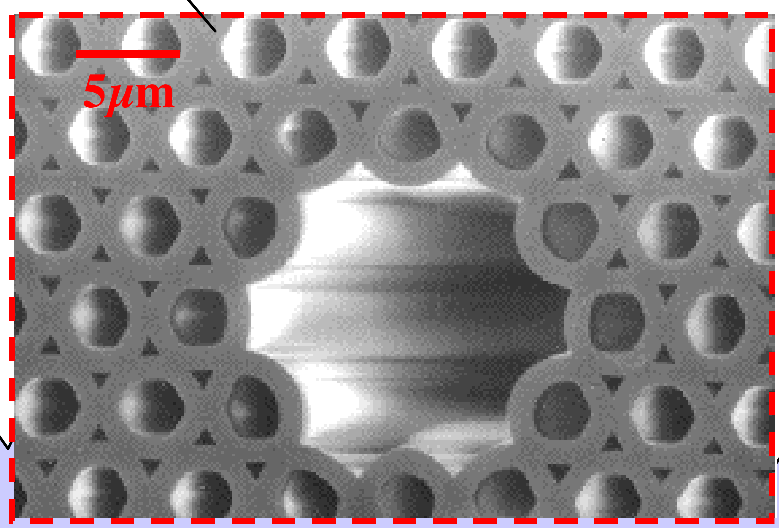
[Yeh *et al.*, 1978]

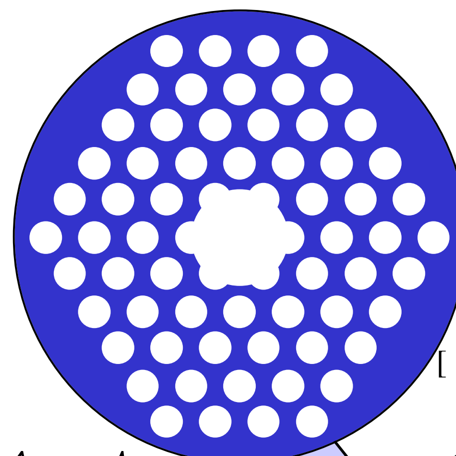


+ omnidirectional
= OmniGuides

silica

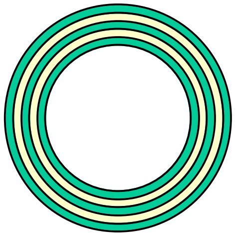
[R. F. Cregan
et al.,
Science **285**,
1537 (1999)]



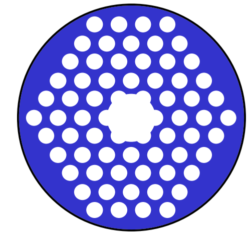


PCF

[Knight *et al.*, 1998]



Breaking the Glass Ceiling:
Hollow-core Bandgap Fibers



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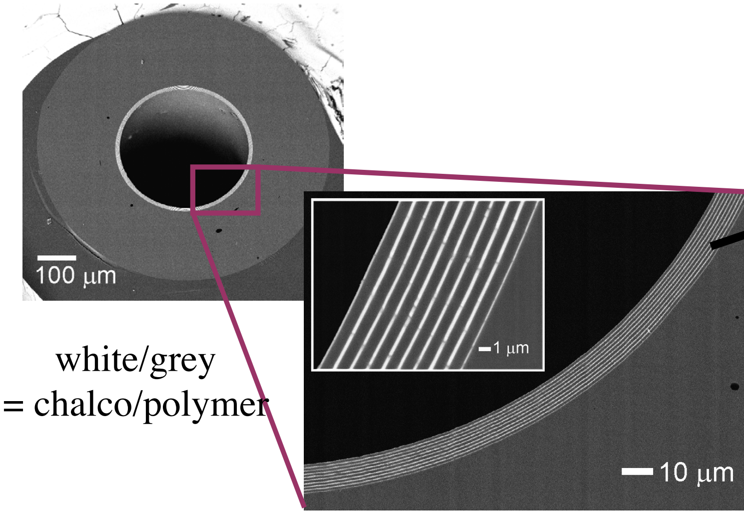
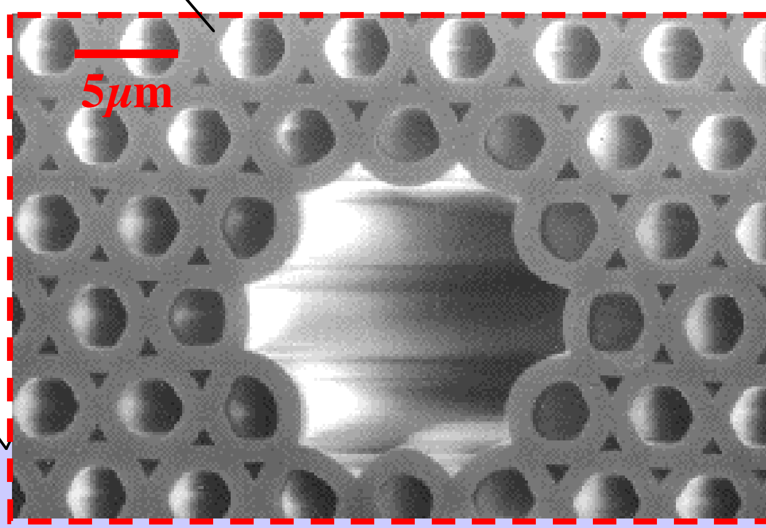


white/grey
= chalco/polymer

Guiding @ 10.6μm
(high-power CO_2 lasers)
loss < 1 dB/m
(material loss ~ 10^4 dB/m)
[Temelkuran *et al.*,
Nature **420**, 650 (2002)]

silica

[R. F. Cregan
et al.,
Science **285**,
1537 (1999)]

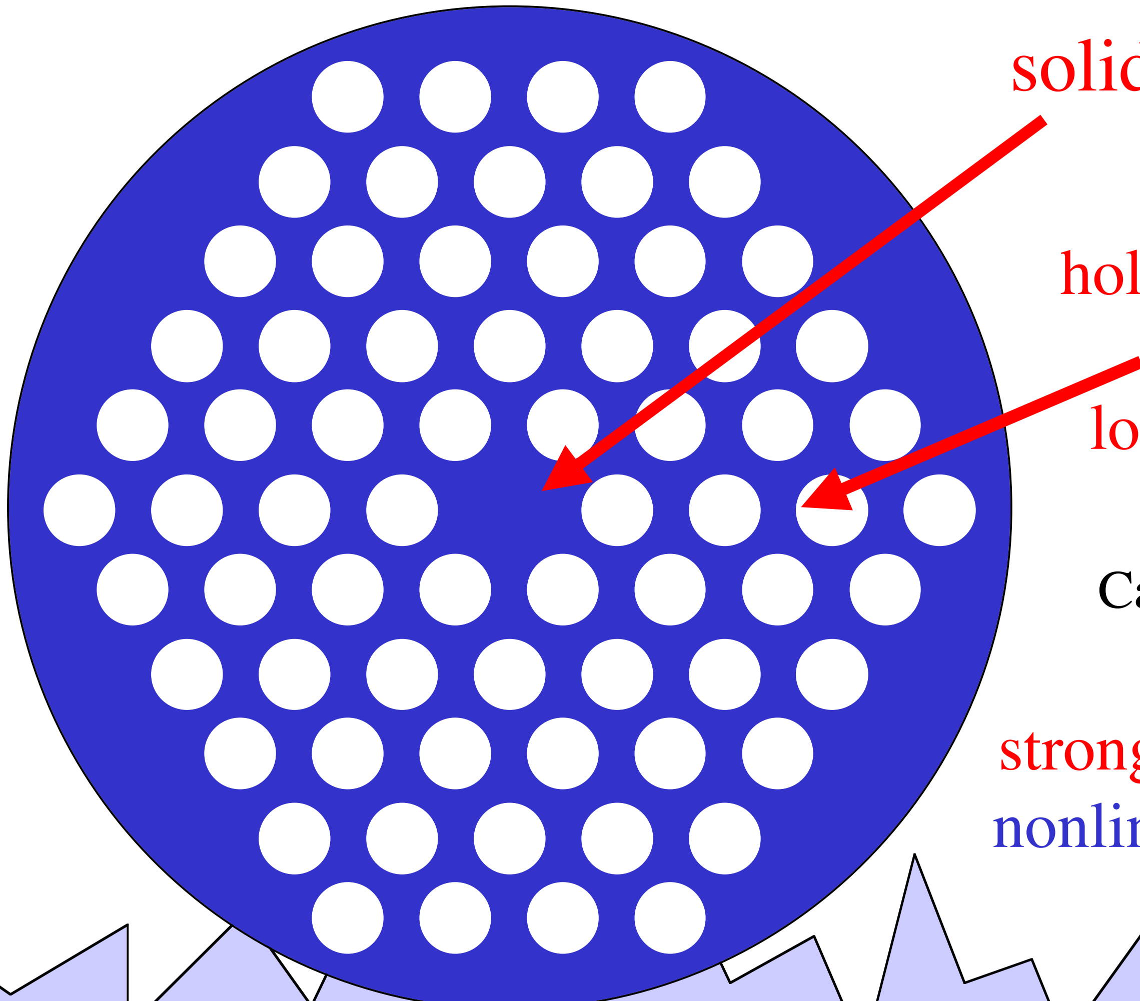


Guiding @ 1.55μm
loss ~ 13dB/km

[Smith, *et al.*,
Nature **424**, 657 (2003)]

OFC 2004: 1.7dB/km
BlazePhotonics

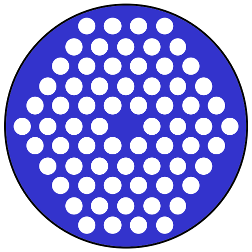
Breaking the Glass Ceiling II:
Solid-core Holey Fibers



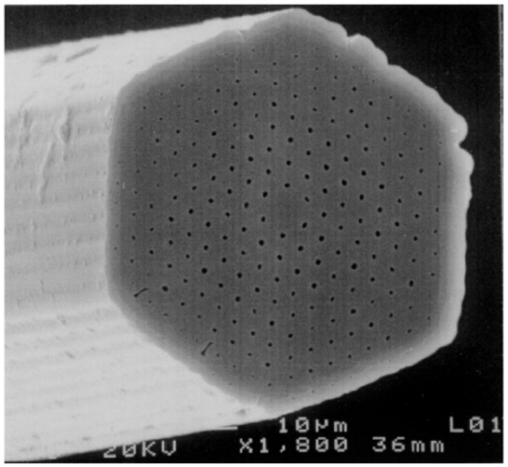
Can have much higher contrast than doped silica…

strong confinement = enhanced nonlinearities, birefringence, …

[J. C. Knight *et al.*, *Opt. Lett.* **21**, 1547 (1996)]



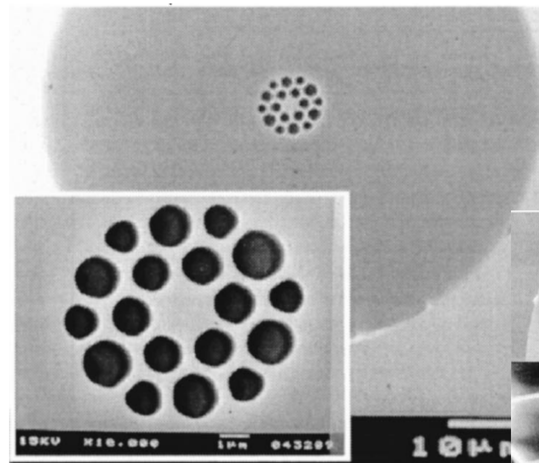
Breaking the Glass Ceiling II:
Solid-core Holey Fibers



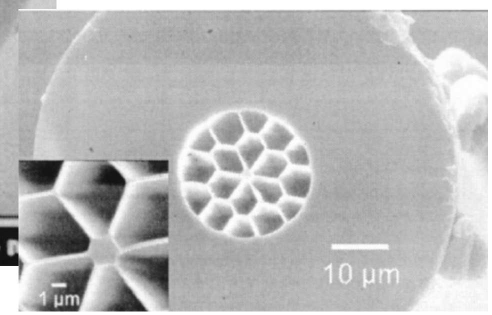
endlessly single-mode

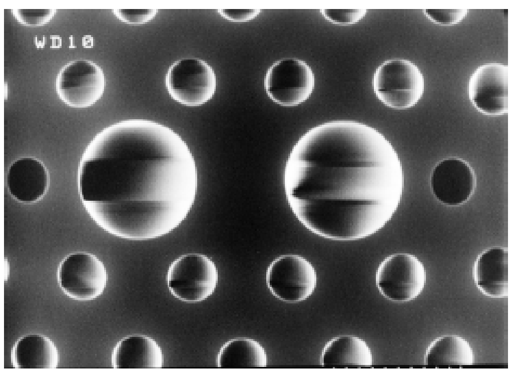
[T. A. Birks *et al*., *Opt. Lett.* **22**, 961 (1997)]

nonlinear fibers



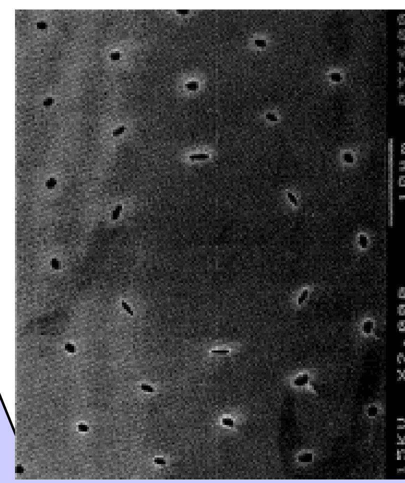
[Wadsworth *et al.*, *JOSA B* **19**, 2148 (2002)]





polarization -maintaining

[K. Suzuki, *Opt. Express* **9**, 676 (2001)]



low-contrast *linear* fiber (large area)

[J. C. Knight *et al.*, *Elec. Lett.* **34**, 1347 (1998)]

Outline

- What are these fibers (and why should I care)?
- The guiding mechanisms: index-guiding and band gaps
- Finding the guided modes
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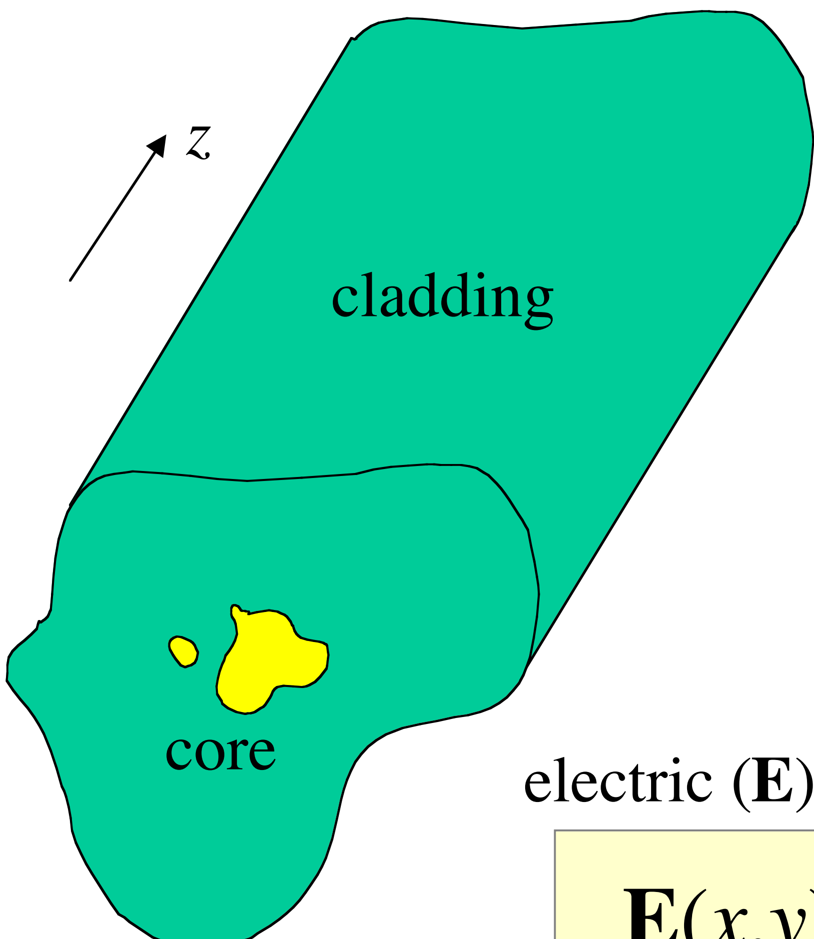
Universal Truths: Conservation Laws

an arbitrary-shaped fiber



(1) Linear, time-invariant system:

(nonlinearities are small correction)

frequency ω is conserved

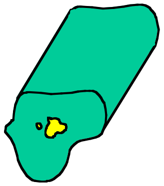
(2) z-invariant system:

(bends *etc*. are small correction)

wavenumber β is conserved

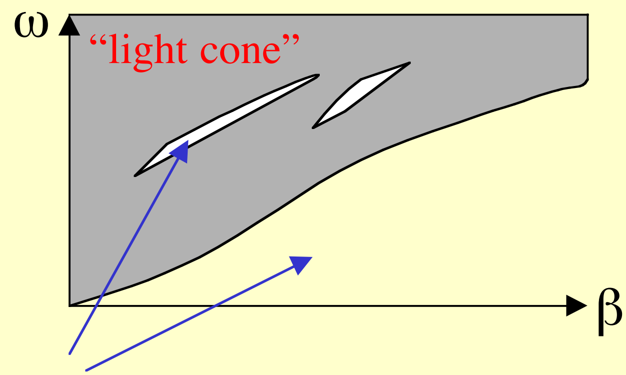
electric (**E**) and magnetic (**H**) fields can be chosen:

$$\mathbf{E}(x,y)\, e^{i(\beta z - \omega t)}, \quad \mathbf{H}(x,y)\, e^{i(\beta z - \omega t)}$$



Sequence of Computation

1. Plot all solutions of infinite cladding as ω vs. β



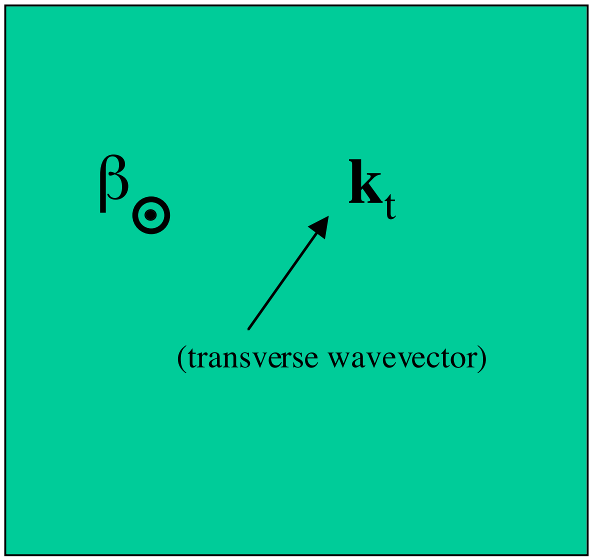
empty spaces (gaps): guiding possibilities

2. Core introduces new states in empty spaces — plot ω(β) dispersion relation

3. Compute other stuff…

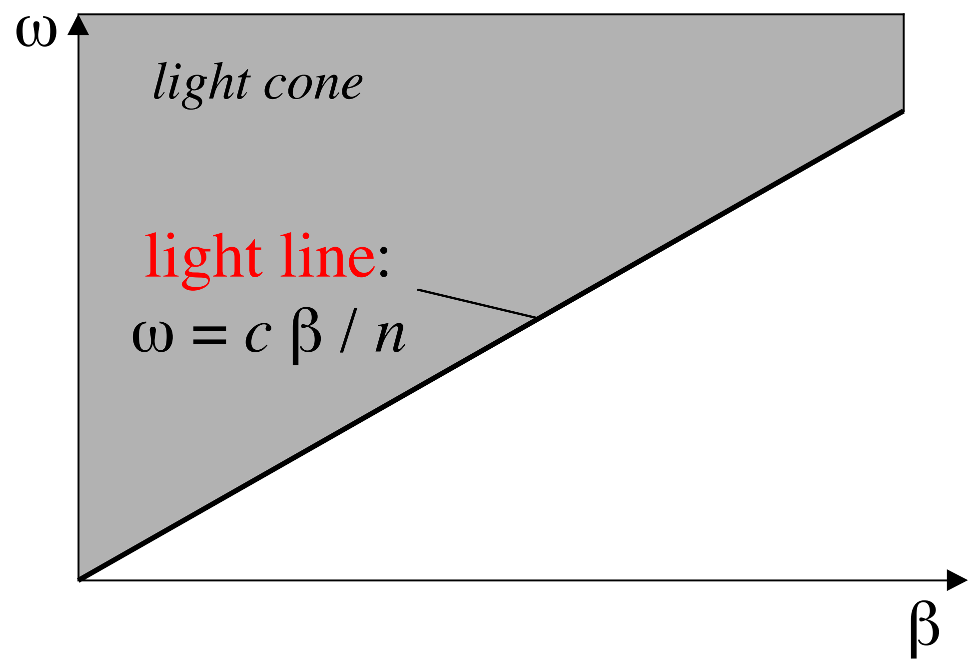
Conventional Fiber: Uniform Cladding

uniform cladding, index *n*



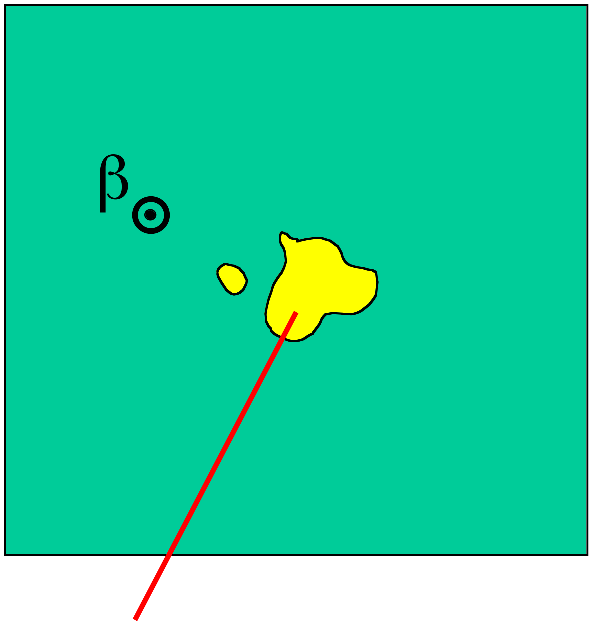
$$\omega = \frac{c}{n}\sqrt{\beta^2 + \left|\mathbf{k}_t\right|^2}$$

$$\geq \frac{c\beta}{n}$$



Conventional Fiber: Uniform Cladding

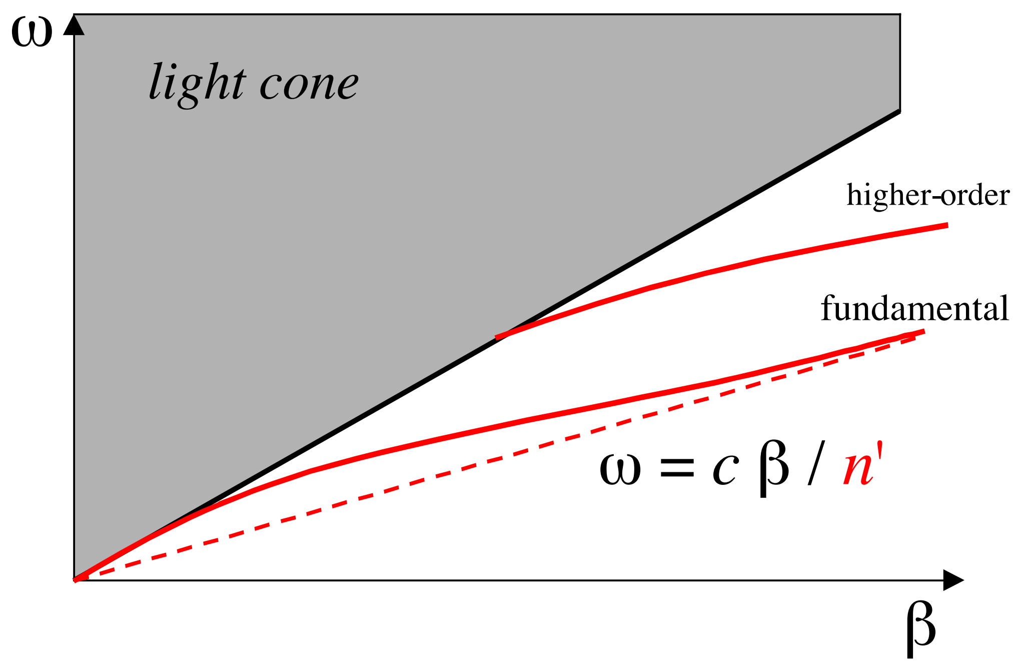
uniform cladding, index *n*



core with higher index *n'*
pulls down
index-guided mode(s)

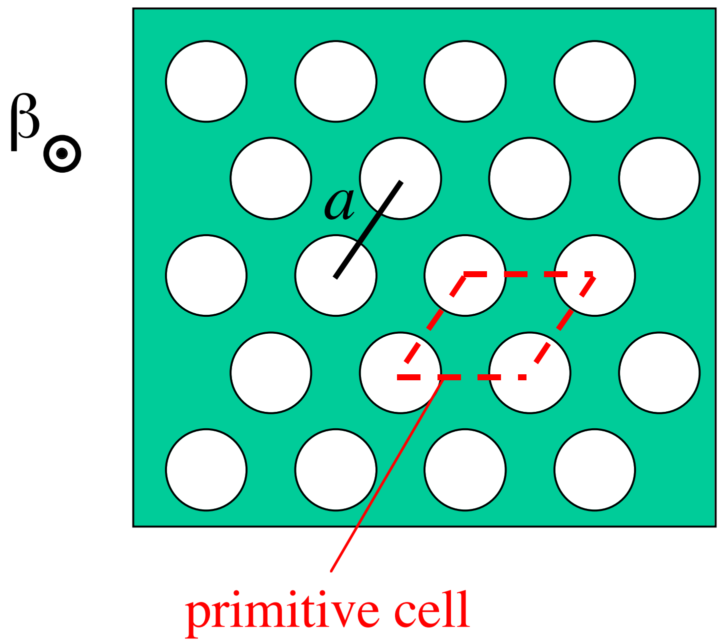
$$\omega = \frac{c}{n}\sqrt{\beta^2 + |\mathbf{k}_t|^2}$$

$$\geq \frac{c\beta}{n}$$



PCF: Periodic Cladding

periodic cladding $\varepsilon(x,y)$



Bloch's Theorem for periodic systems:

fields can be written:

$$\mathbf{E}(x,y)\, e^{i(\beta z + \mathbf{k}_t \mathbf{x}_t - \omega t)}, \quad \mathbf{H}(x,y)\, e^{i(\beta z + \mathbf{k}_t \mathbf{x}_t - \omega t)}$$

periodic functions on primitive cell

transverse (xy) Bloch wavevector $\mathbf{k}_t$

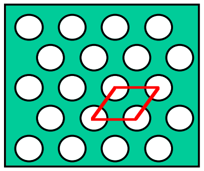
satisfies eigenproblem (Hermitian if lossless)

$$\nabla_{\mathbf{k}_t,\beta} \times \frac{1}{\varepsilon} \nabla_{\mathbf{k}_t,\beta} \times \mathbf{H} = \frac{\omega^2}{c^2} \mathbf{H}$$

constraint: $\nabla_{\mathbf{k}_t,\beta} \cdot \mathbf{H} = 0$

where:

$$\nabla_{\mathbf{k}_t,\beta} = \nabla + i\mathbf{k}_t + i\beta\hat{\mathbf{z}}$$



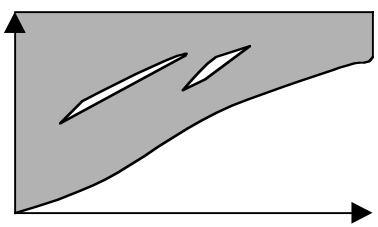
PCF: Cladding Eigensolution

Finite cell ➔ *discrete* eigenvalues ω_n

Want to solve for $\omega_n(\mathbf{k}_t, \beta)$, & plot vs. β for "all" n, $\mathbf{k}_t$

$$\nabla_{\mathbf{k}_t,\beta} \times \frac{1}{\varepsilon} \nabla_{\mathbf{k}_t,\beta} \times \mathbf{H}_n = \frac{\omega_n^2}{c^2} \mathbf{H}_n$$

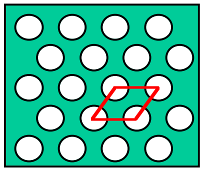
constraint: $\nabla_{\mathbf{k}_t,\beta} \cdot \mathbf{H} = 0$



where: $\nabla_{\mathbf{k}_t,\beta} = \nabla + i\mathbf{k}_t + i\beta\hat{\mathbf{z}}$

$$\mathbf{H}(x,y)\, e^{i(\beta z + \mathbf{k}_t \mathbf{x}_t - \omega t)}$$

1. Limit range of $\mathbf{k}_t$: irreducible Brillouin zone
2. Limit degrees of freedom: expand $\mathbf{H}$ in finite basis
3. Efficiently solve eigenproblem: iterative methods

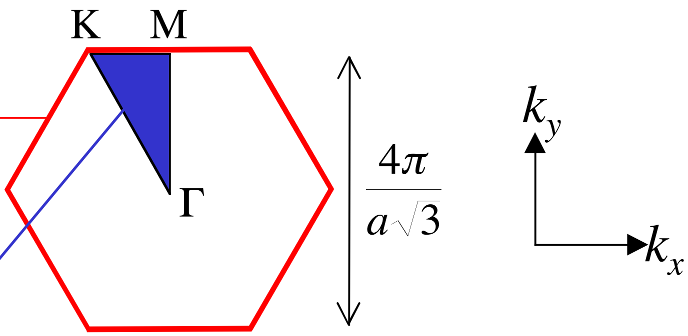


PCF: Cladding Eigensolution

1. Limit range of $\mathbf{k}_t$: irreducible Brillouin zone

—Bloch's theorem: solutions are periodic in $\mathbf{k}_t$

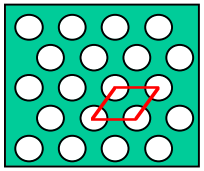
first Brillouin zone = minimum $|\mathbf{k}_t|$ "primitive cell"



irreducible Brillouin zone: reduced by symmetry

2. Limit degrees of freedom: expand **H** in finite basis

3. Efficiently solve eigenproblem: iterative methods



PCF: Cladding Eigensolution

1. Limit range of $\mathbf{k}_t$: irreducible Brillouin zone

2. Limit degrees of freedom: expand **H** in finite basis

— must satisfy constraint: $\nabla_{\mathbf{k}_t,\beta} \cdot \mathbf{H} = 0$

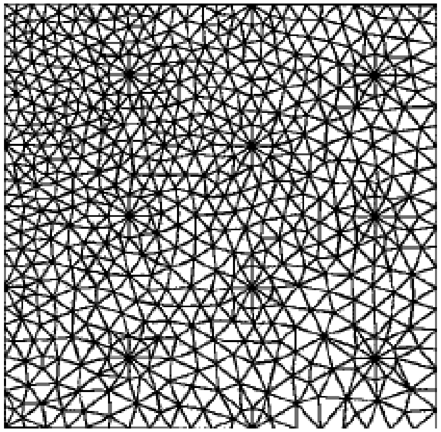
Planewave (FFT) basis

$$\mathbf{H}(\mathbf{x}_t) = \sum_{\mathbf{G}} \mathbf{H}_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{x}_t}$$

constraint: $\mathbf{H}_{\mathbf{G}} \cdot (\mathbf{G} + \mathbf{k} + \beta\hat{\mathbf{z}}) = 0$

uniform "grid," periodic boundaries, simple code, O(N log N)

Finite-element basis



[figure: Peyrilloux *et al.*, *J. Lightwave Tech.* **21**, 536 (2003)]

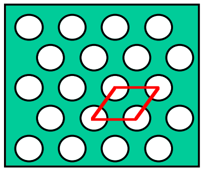
constraint, boundary conditions:

Nédélec elements

[Nédélec, *Numerische Math.* **35**, 315 (1980)]

nonuniform mesh, more arbitrary boundaries, complex code & mesh, O(N)

3. Efficiently solve eigenproblem: iterative methods



PCF: Cladding Eigensolution

1. Limit range of $\mathbf{k}_t$: irreducible Brillouin zone

2. Limit degrees of freedom: expand **H** in finite basis (N)

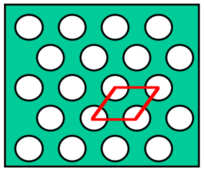
$$|\mathbf{H}\rangle = \mathbf{H}(\mathbf{x}_t) = \sum_{m=1}^{N} h_m \mathbf{b}_m(\mathbf{x}_t)$$

solve: $\hat{A}|\mathbf{H}\rangle = \omega^2 |\mathbf{H}\rangle$

finite matrix problem: $Ah = \omega^2 Bh$

$$\langle \mathbf{f} | \mathbf{g} \rangle = \int \mathbf{f}^* \cdot \mathbf{g} \qquad A_{m\ell} = \langle \mathbf{b}_m | \hat{A} | \mathbf{b}_\ell \rangle \qquad B_{m\ell} = \langle \mathbf{b}_m | \mathbf{b}_\ell \rangle$$

3. Efficiently solve eigenproblem: iterative methods



PCF: Cladding Eigensolution

1. Limit range of $\mathbf{k}_t$: irreducible Brillouin zone
2. Limit degrees of freedom: expand **H** in finite basis
3. Efficiently solve eigenproblem: iterative methods

$$Ah = \omega^2 Bh$$

Slow way: compute A & B, ask LAPACK for eigenvalues

— requires $O(N^2)$ storage, $O(N^3)$ time

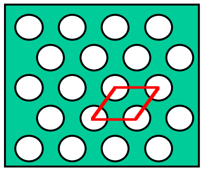
Faster way:

— start with *initial guess* eigenvector h_0

— *iteratively* improve

— $O(Np)$ storage, ~ $O(Np^2)$ time for p eigenvectors

(p smallest eigenvalues)



PCF: Cladding Eigensolution

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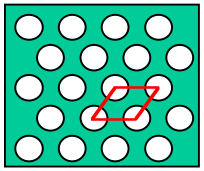
$$Ah = \omega^2 Bh$$

Many iterative methods:

— Arnoldi, Lanczos, Davidson, Jacobi-Davidson, …, Rayleigh-quotient minimization



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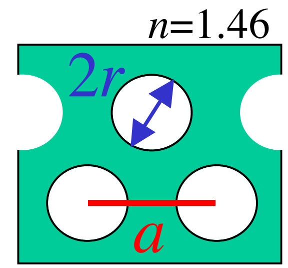
for Hermitian matrices, smallest eigenvalue ω_0 minimizes:

"variational theorem"

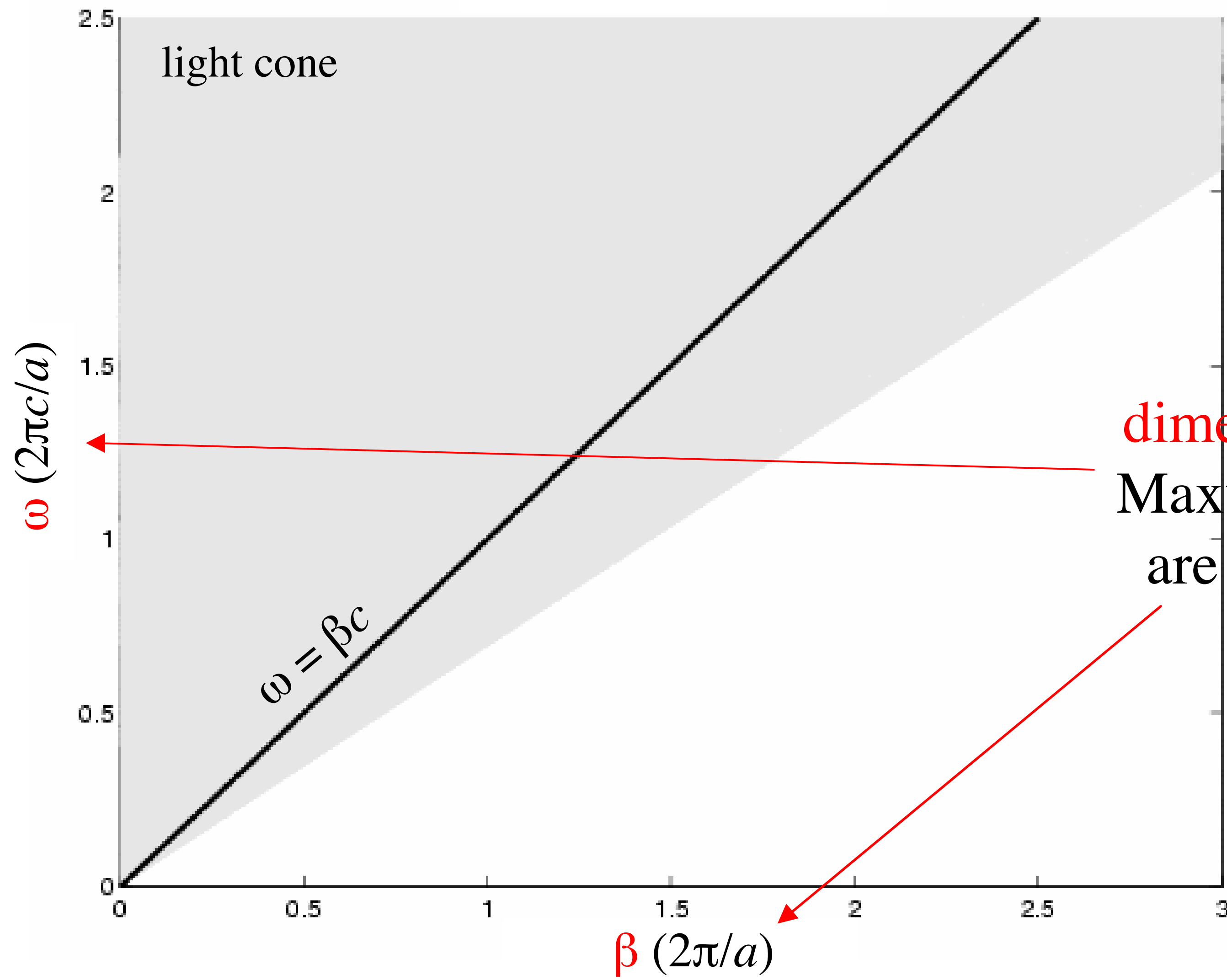
$$\omega_0^2 = \min_h \frac{h' Ah}{h' Bh}$$

minimize by conjugate-gradient, (or multigrid, *etc.*)

PCF: Holey Silica Cladding

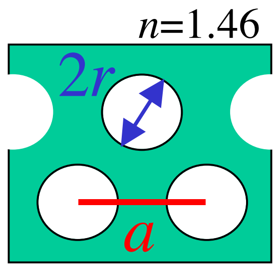


$r = 0.1a$

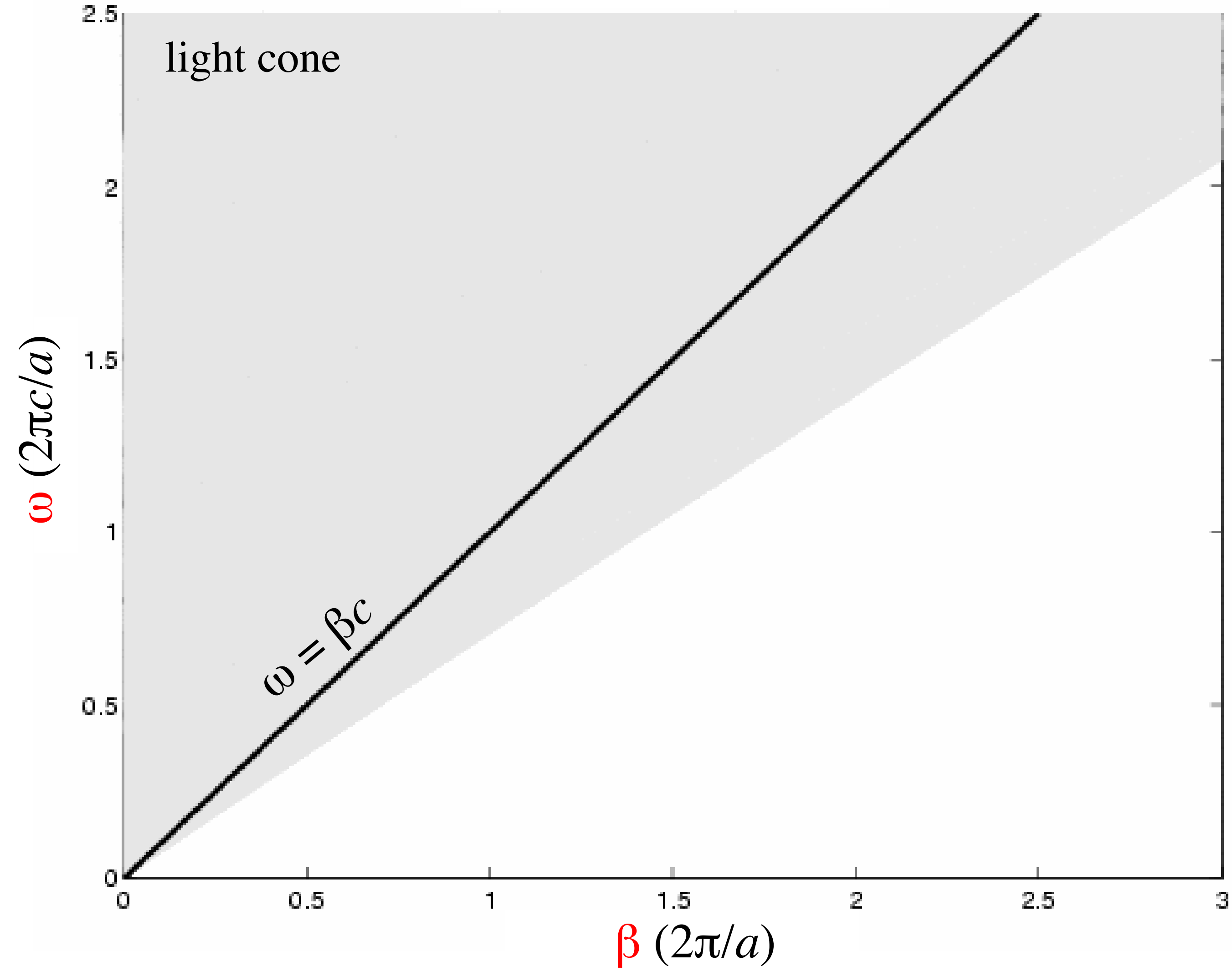


dimensionless units: Maxwell's equations are scale-invariant

PCF: Holey Silica Cladding



$r = 0.17717a$



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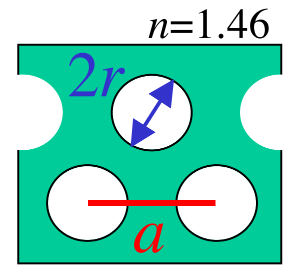
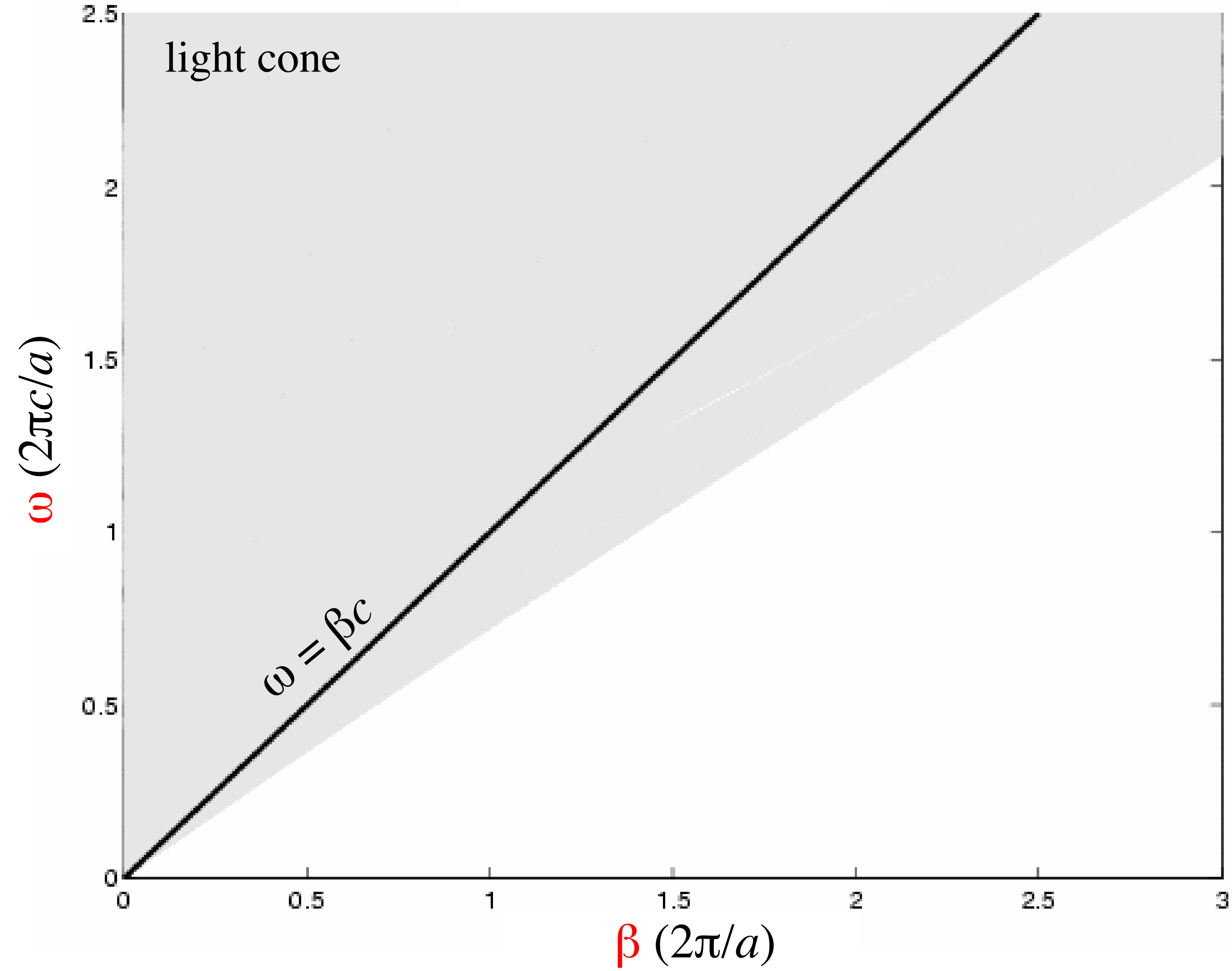
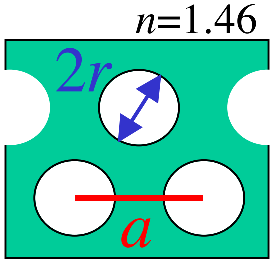


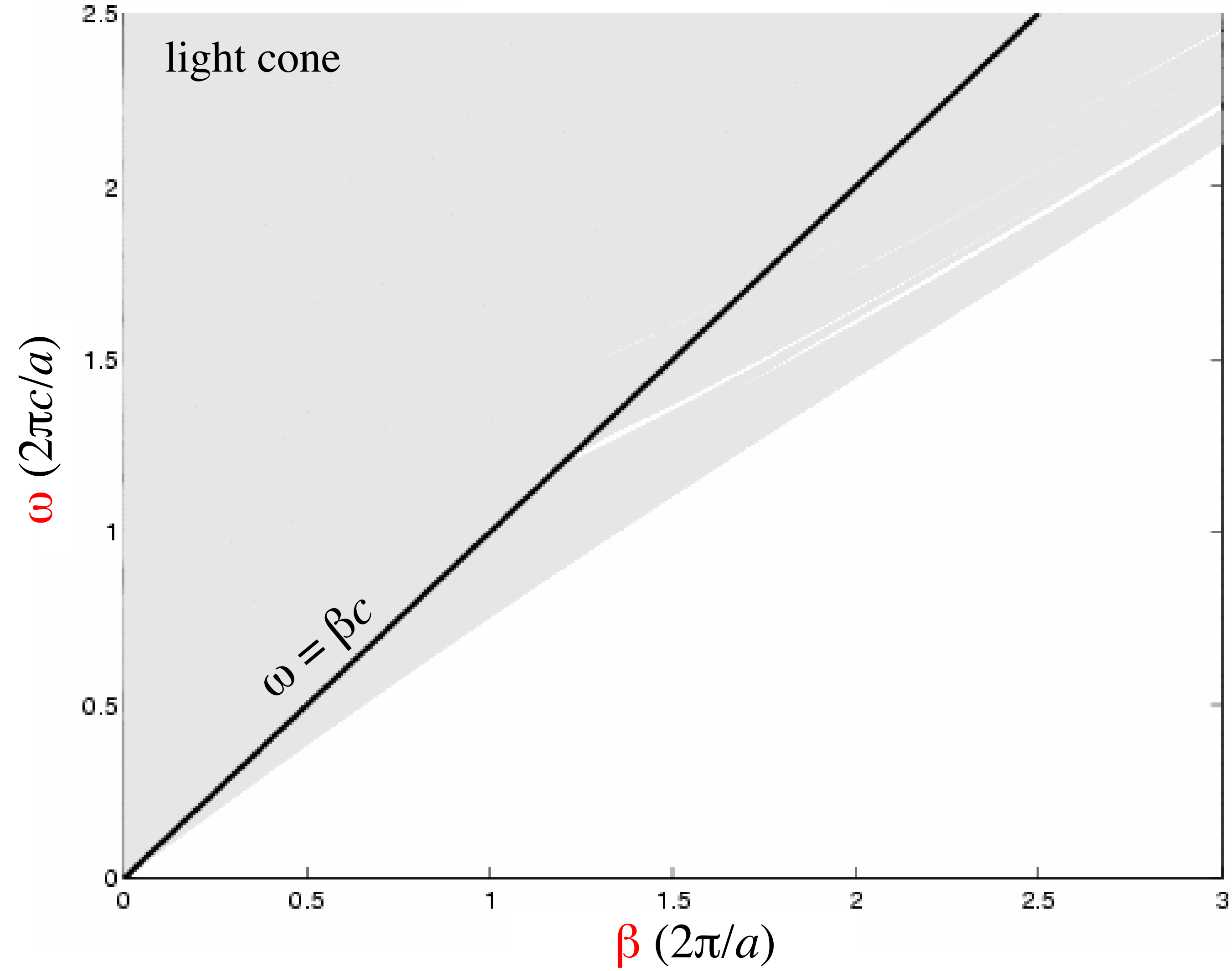
$r = 0.22973a$



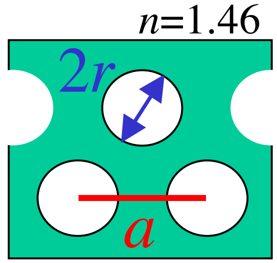
PCF: Holey Silica Cladding



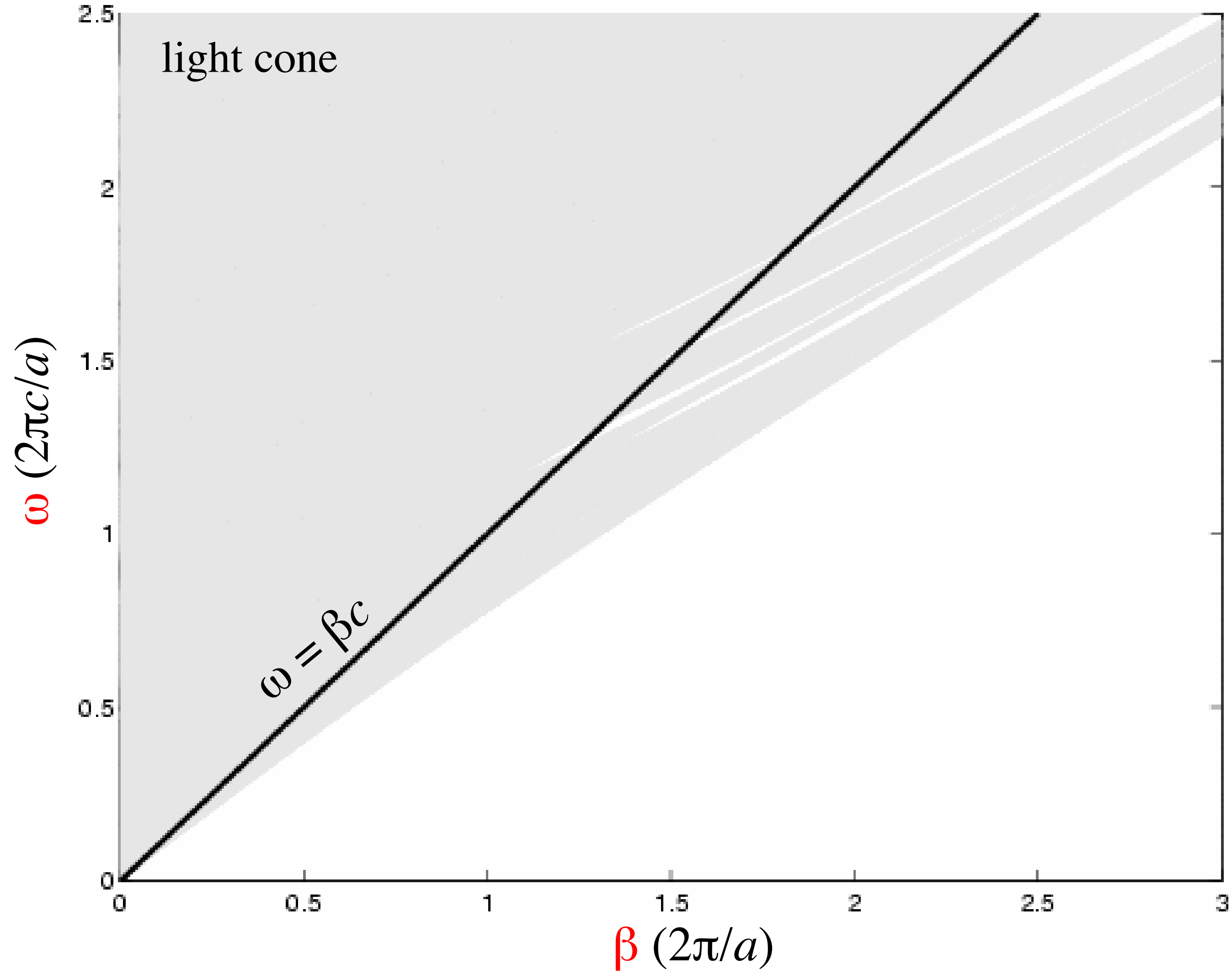
$r = 0.30912a$



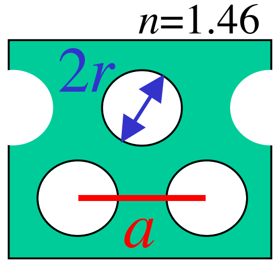
PCF: Holey Silica Cladding



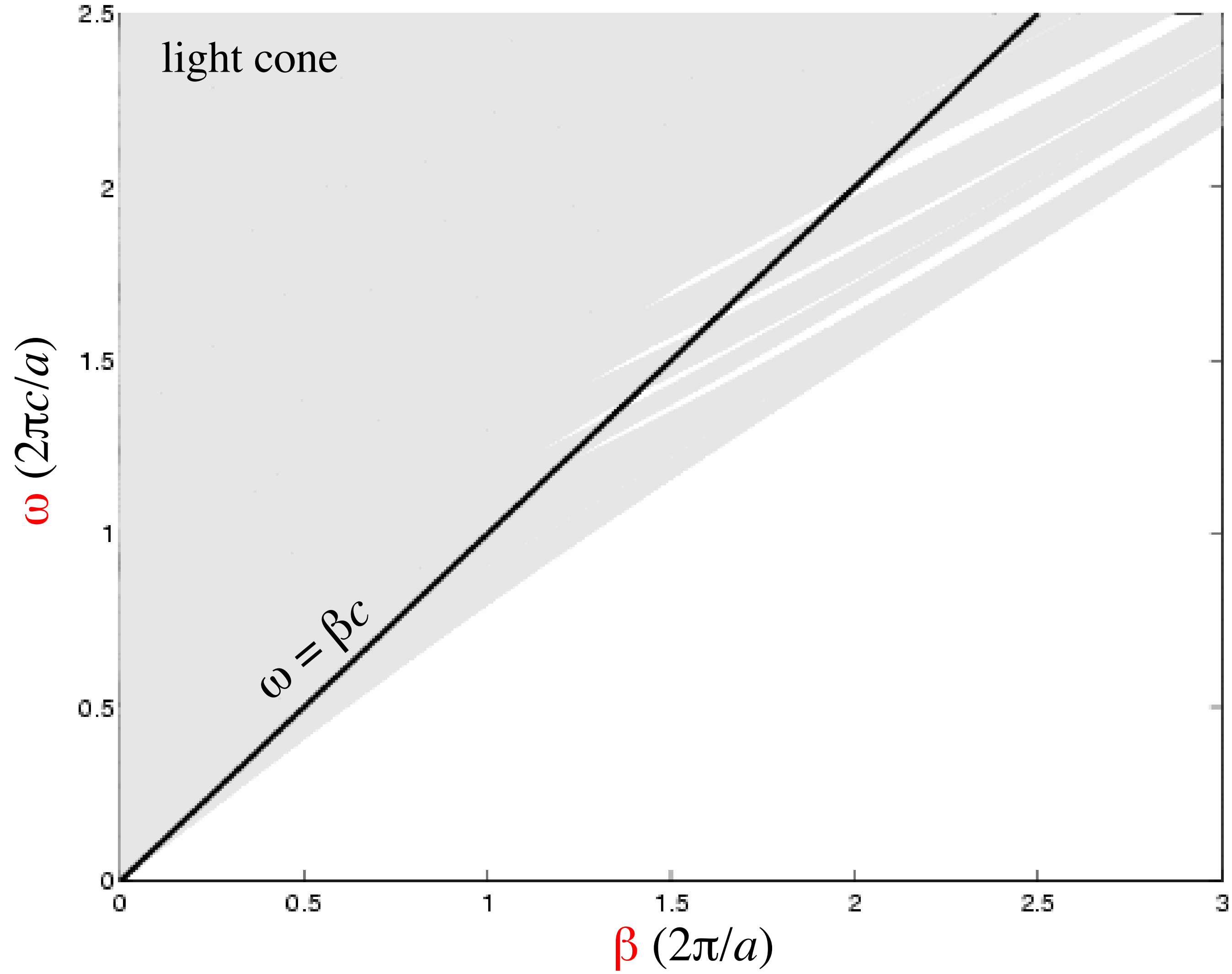
$r = 0.34197a$



PCF: Holey Silica Cladding



$r = 0.37193a$



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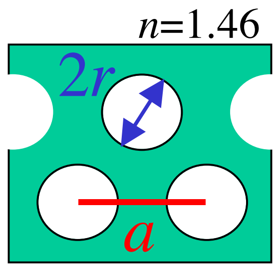
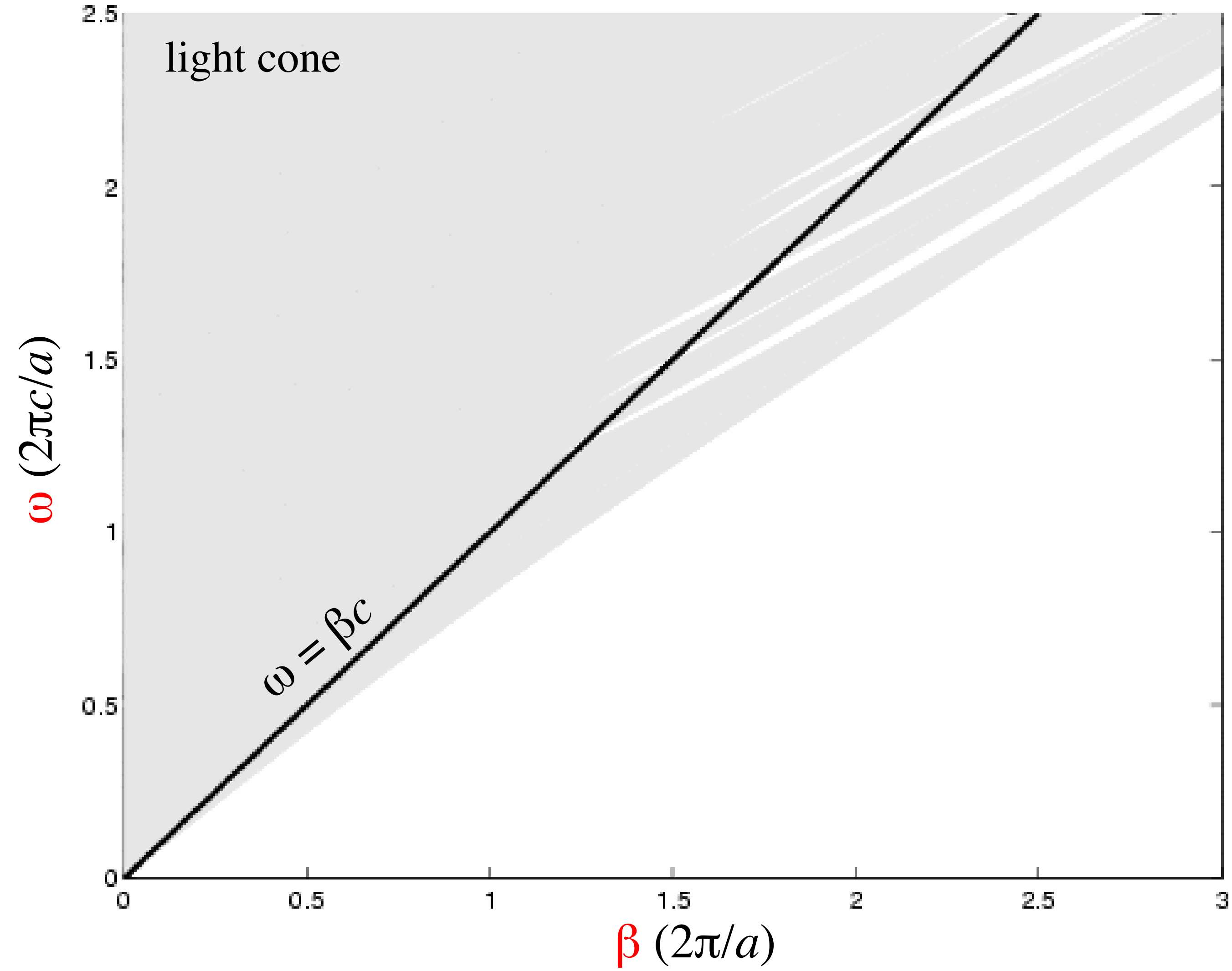
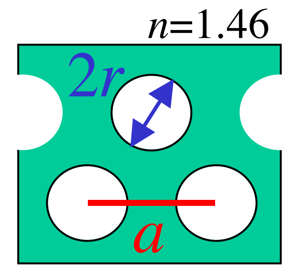


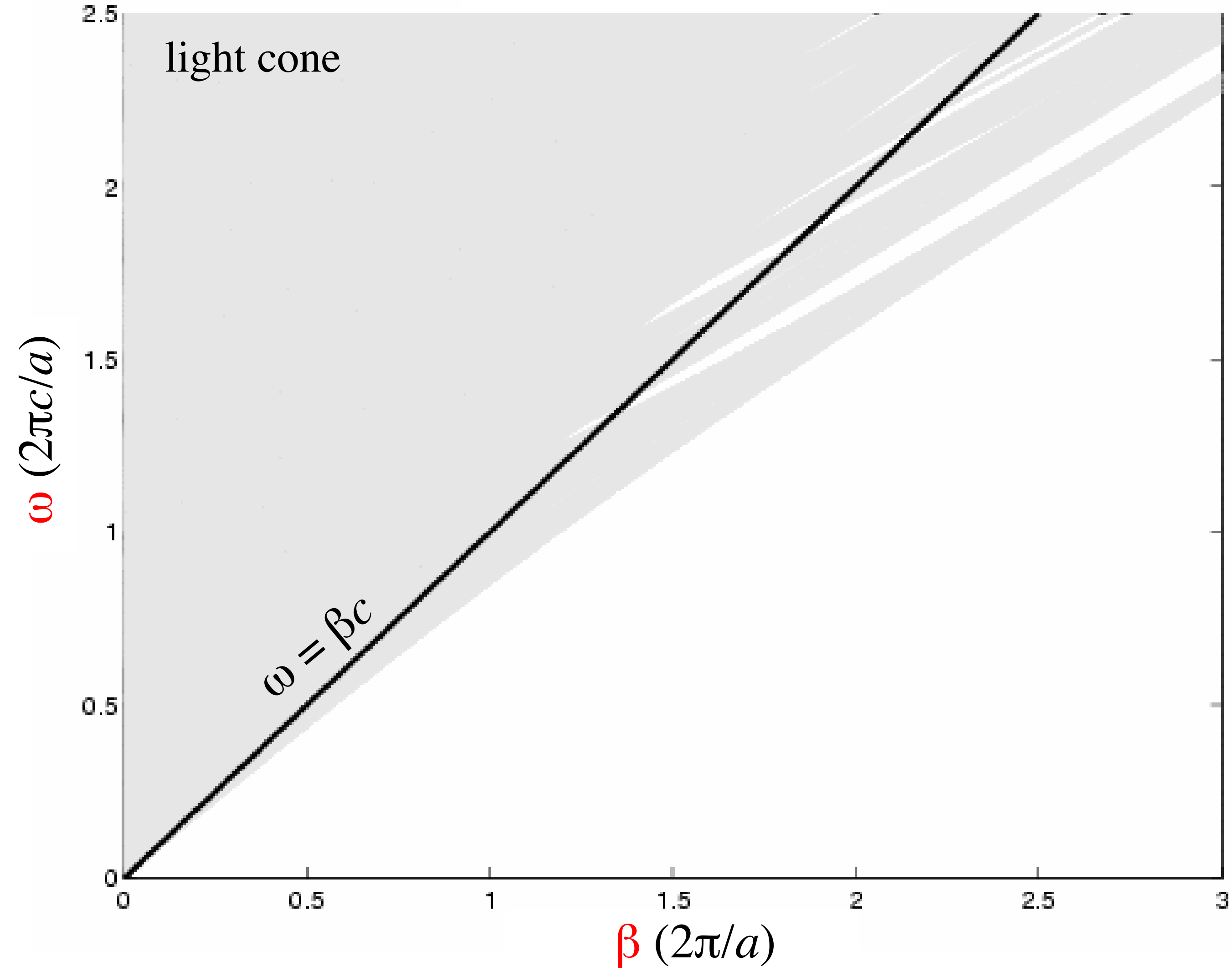
$r = 0.4a$



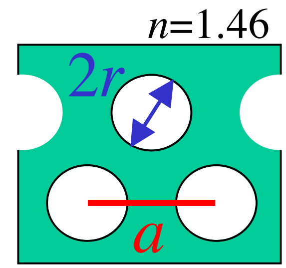
PCF: Holey Silica Cladding



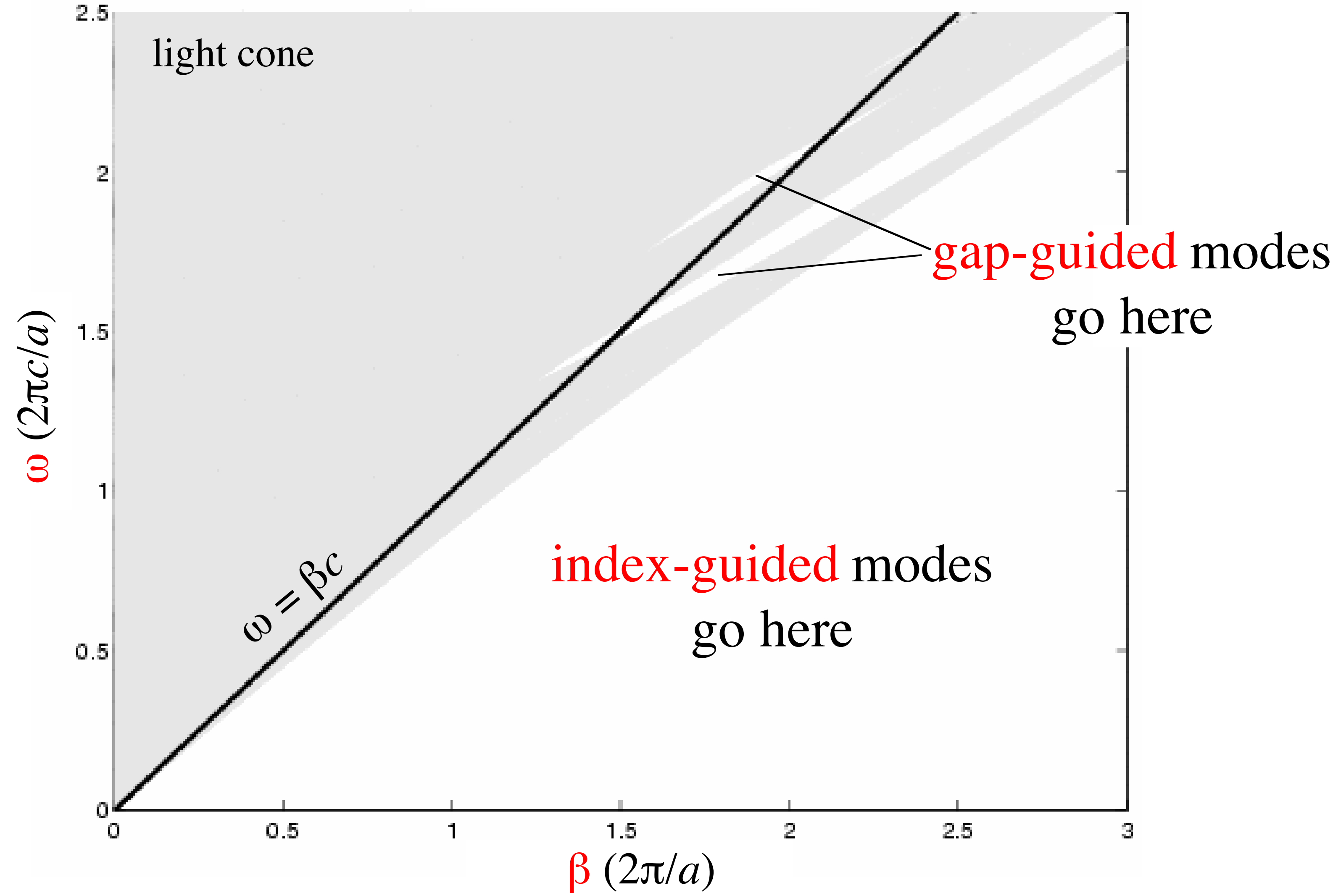
$r = 0.42557a$



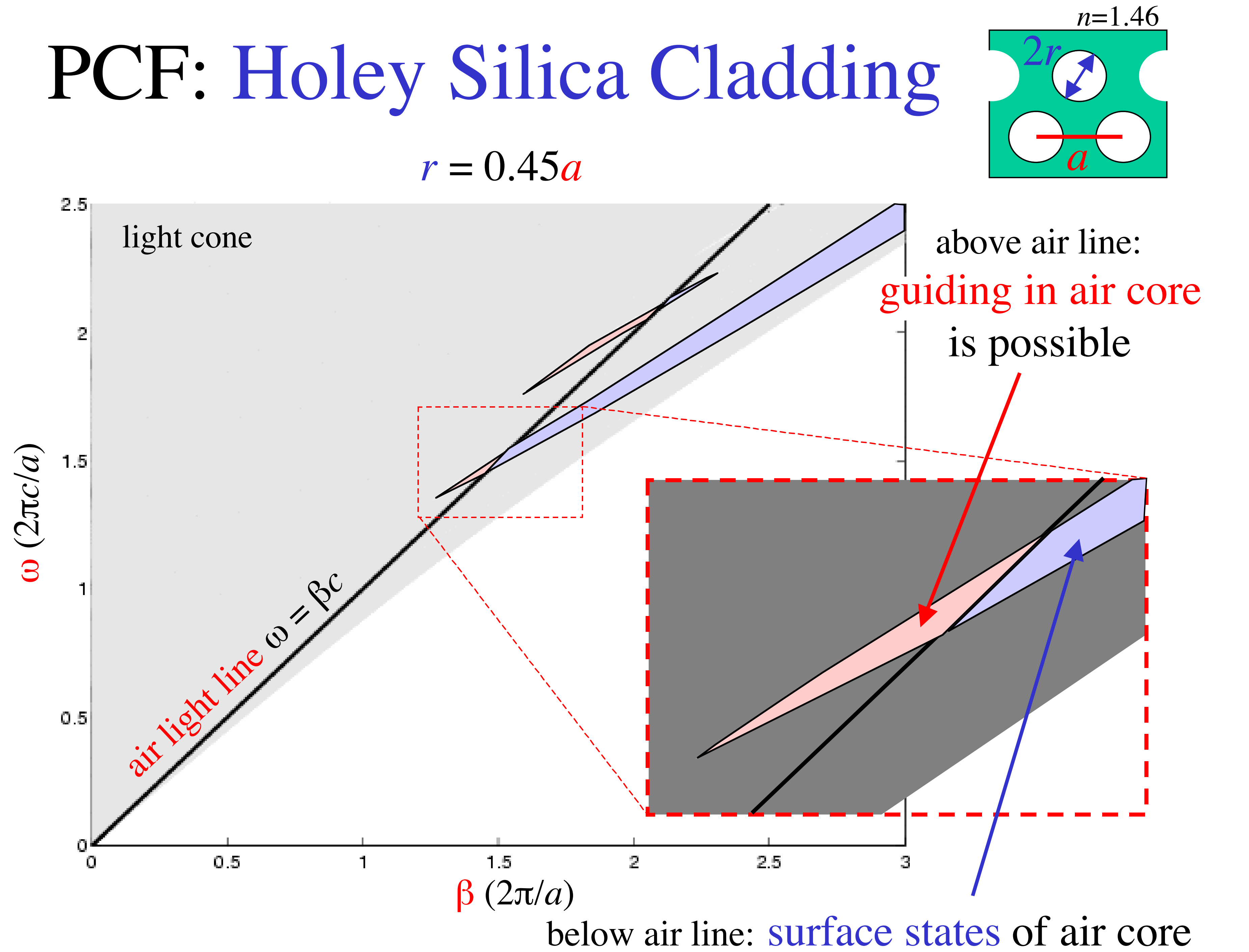
PCF: Holey Silica Cladding

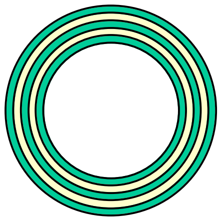


$r = 0.45a$



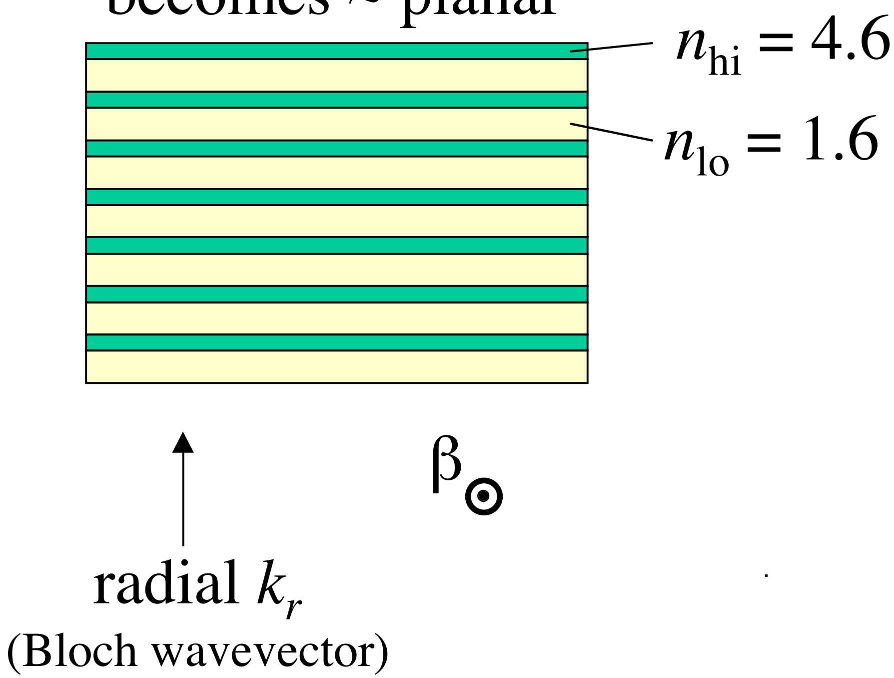
PCF: Holey Silica Cladding

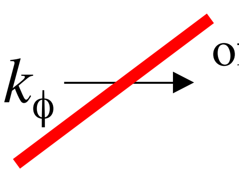




Bragg Fiber Cladding

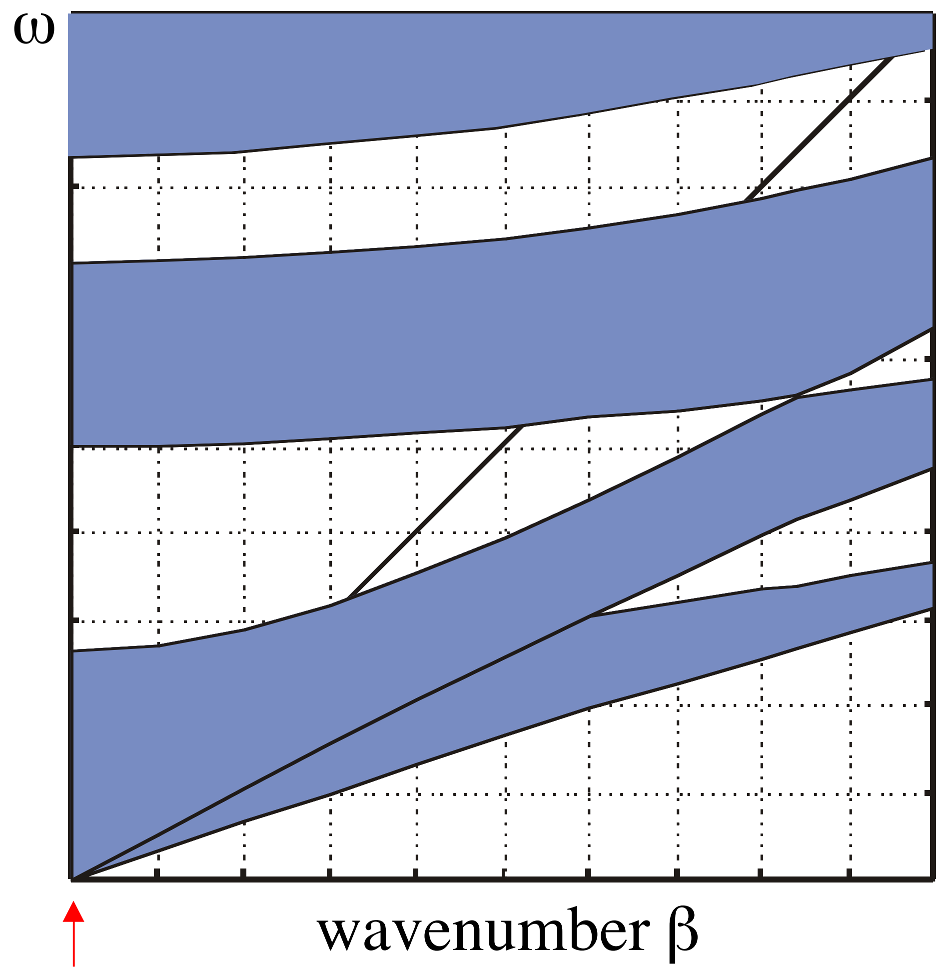
at large radius,
becomes ~ planar



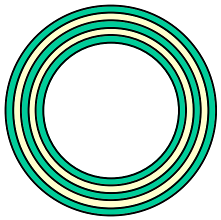


0 by conservation
of angular momentum

Bragg fiber gaps (1d eigenproblem)

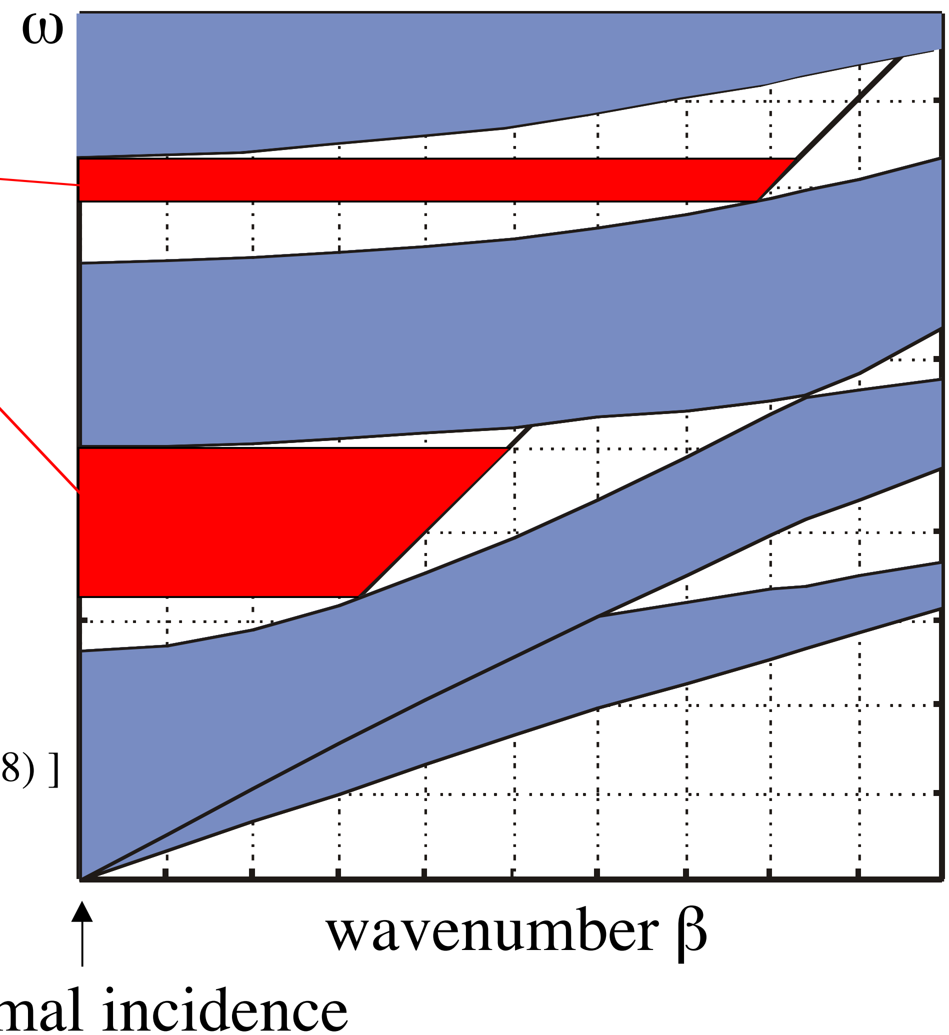


β = 0: normal incidence



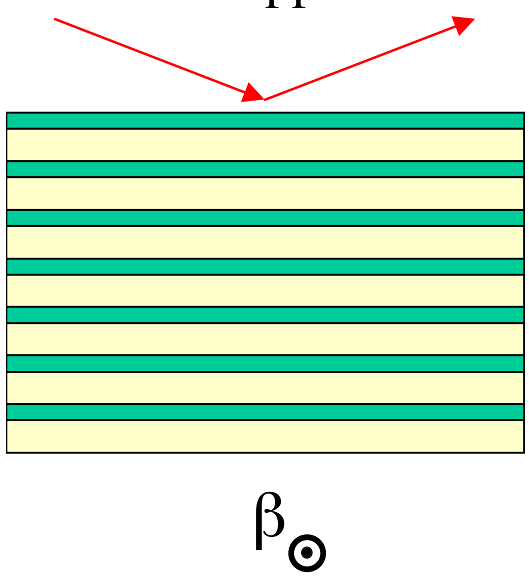
Omnidirectional Cladding

Bragg fiber gaps (1d eigenproblem)



omnidirectional
(planar) reflection

e.g. light from
fluorescent sources
is trapped

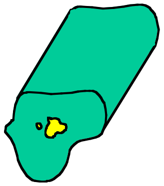


for n_{hi} / n_{lo}
big enough
and $n_{lo} > 1$

[J. N. Winn *et al*,
Opt. Lett. **23**, 1573 (1998)]

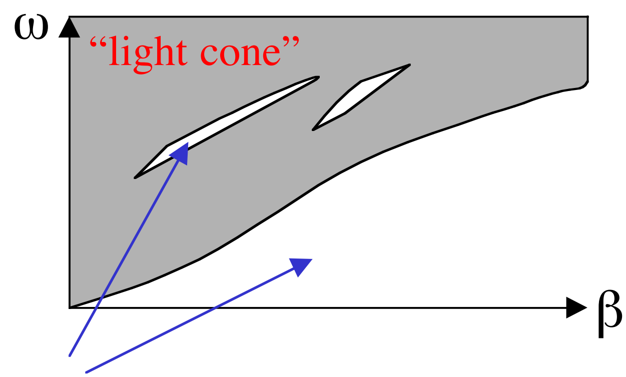
Outline

- What are these fibers (and why should I care)?
- The guiding mechanisms: index-guiding and band gaps
- Finding the guided modes
- Small corrections (with big impacts)



Sequence of Computation

1. Plot all solutions of infinite cladding as ω vs. β



empty spaces (gaps): guiding possibilities

2. Core introduces new states in empty spaces — plot ω(β) dispersion relation

3. Compute other stuff…

Computing Guided (Core) Modes

$$\nabla_\beta \times \frac{1}{\varepsilon} \nabla_\beta \times \mathbf{H}_n = \frac{\omega_n^2}{c^2} \mathbf{H}_n$$

constraint: $\nabla_\beta \cdot \mathbf{H} = 0$

where: $\nabla_\beta = \nabla + i\beta\hat{\mathbf{z}}$

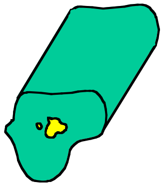
magnetic field = $\mathbf{H}(x,y)\, e^{i(\beta z - \omega t)}$

Same differential equation
as before,
…except no $\mathbf{k}_t$

— can solve the *same* way

New considerations:

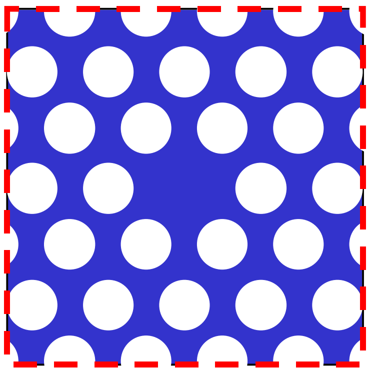
1. Boundary conditions
2. Leakage (finite-size) radiation loss
3. Interior eigenvalues



Computing Guided (Core) Modes

1 Boundary conditions

computational cell



Only care about guided modes:

— exponentially decaying outside core

Effect of boundary cond. decays exponentially

— mostly, boundaries are irrelevant!

periodic (planewave), conducting, absorbing all okay

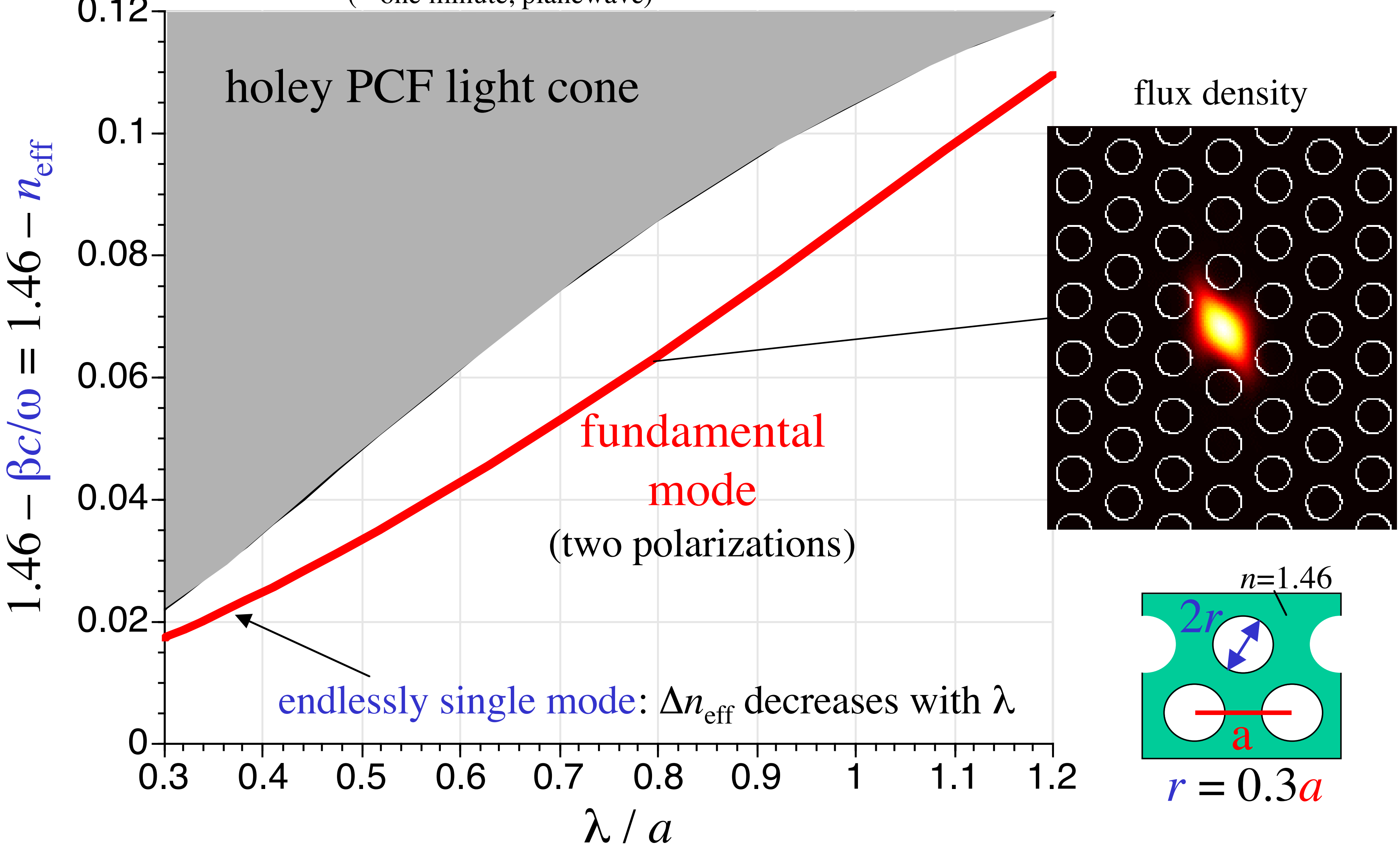
2 Leakage (finite-size) radiation loss

3 Interior eigenvalues

Guided Mode in a Solid Core

small computation: only lowest-ω band!

(~ one minute, planewave)



Fixed-frequency Modes?

Here, we are computing ω(β'),
but we often want β(ω') — λ is specified

No problem!

Just find root of ω(β') – ω', using Newton's method:
(Factor of 3–4 in time.)

$$\beta' \leftarrow \beta' - \frac{\omega - \omega'}{d\omega / d\beta}$$

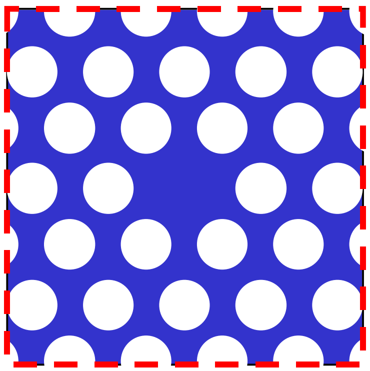
group velocity = power / (energy density)

(*a.k.a.* Hellman-Feynman theorem,
a.k.a. first-order perturbation theory,
a.k.a. "*k*-dot-*p*" theory)

Computing Guided (Core) Modes

1 Boundary conditions

computational cell



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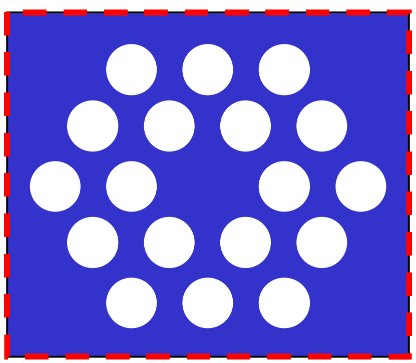


…*except* when we want (small) finite-size losses…

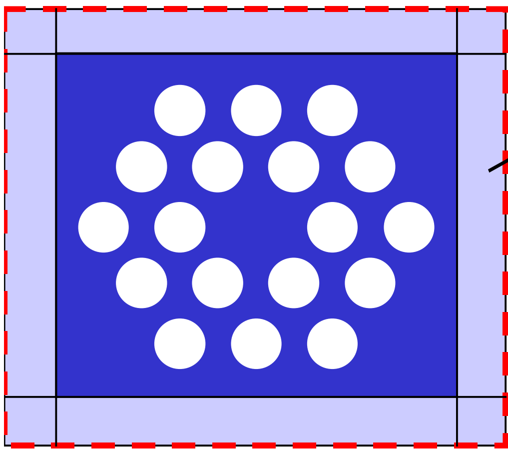
2 Leakage (finite-size) radiation loss

3 Interior eigenvalues

Computing Guided (Core) Modes

1. Boundary conditions

2. Leakage (finite-size) radiation loss



Use PML absorbing boundary layer
perfectly matched layer

[Berenger, *J. Comp. Phys.* **114**, 185 (1994)]

…with iterative method that works for non-Hermitian (dissipative) systems: Jacobi-Davidson, …

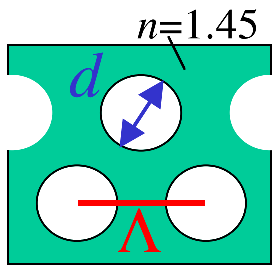
Or imaginary-distance BPM: [Saitoh, *IEEE J. Quantum Elec.* **38**, 927 (2002)]

in imaginary *z*, largest β (fundamental) mode grows exponentially

3. Interior eigenvalues

Computing Guided (Core) Modes

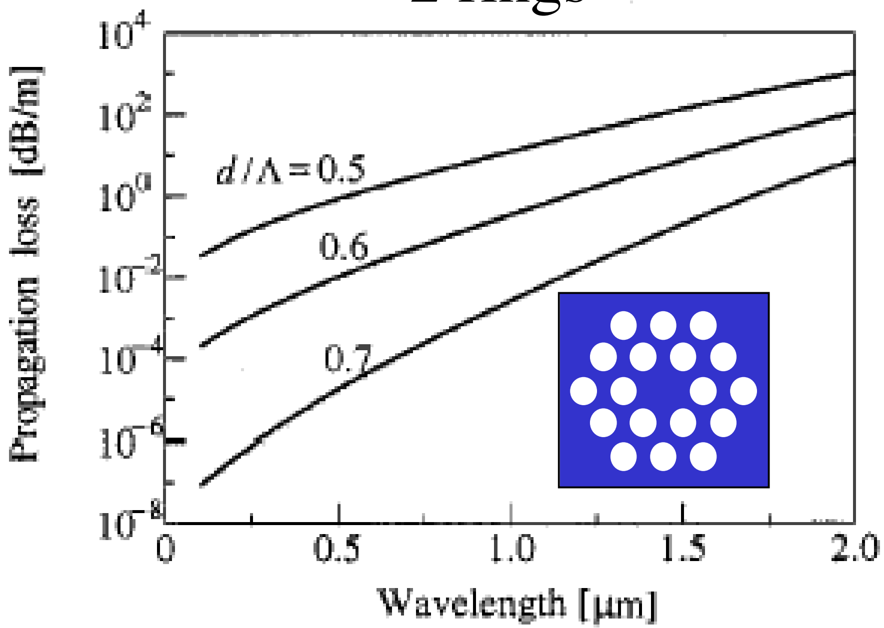
1. Boundary conditions
2. Leakage (finite-size) radiation loss



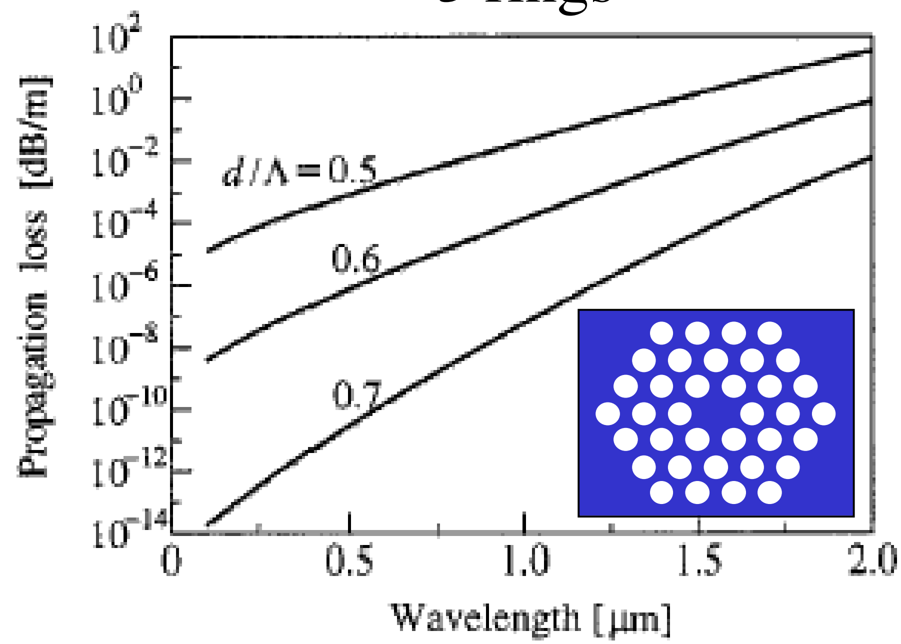
imaginary-distance BPM

[Saitoh, *IEEE J. Quantum Elec*. **38**, 927 (2002)]

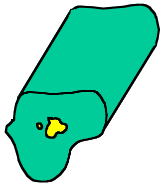
2 rings



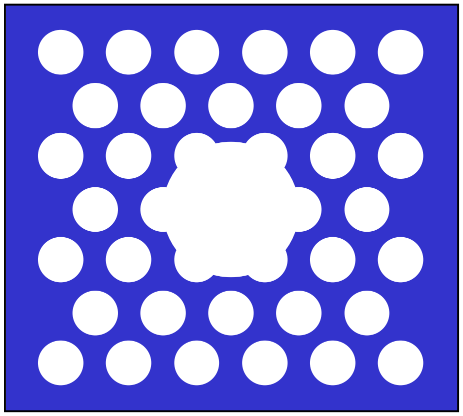
3 rings



3. Interior eigenvalues



Computing Guided (Core) Modes

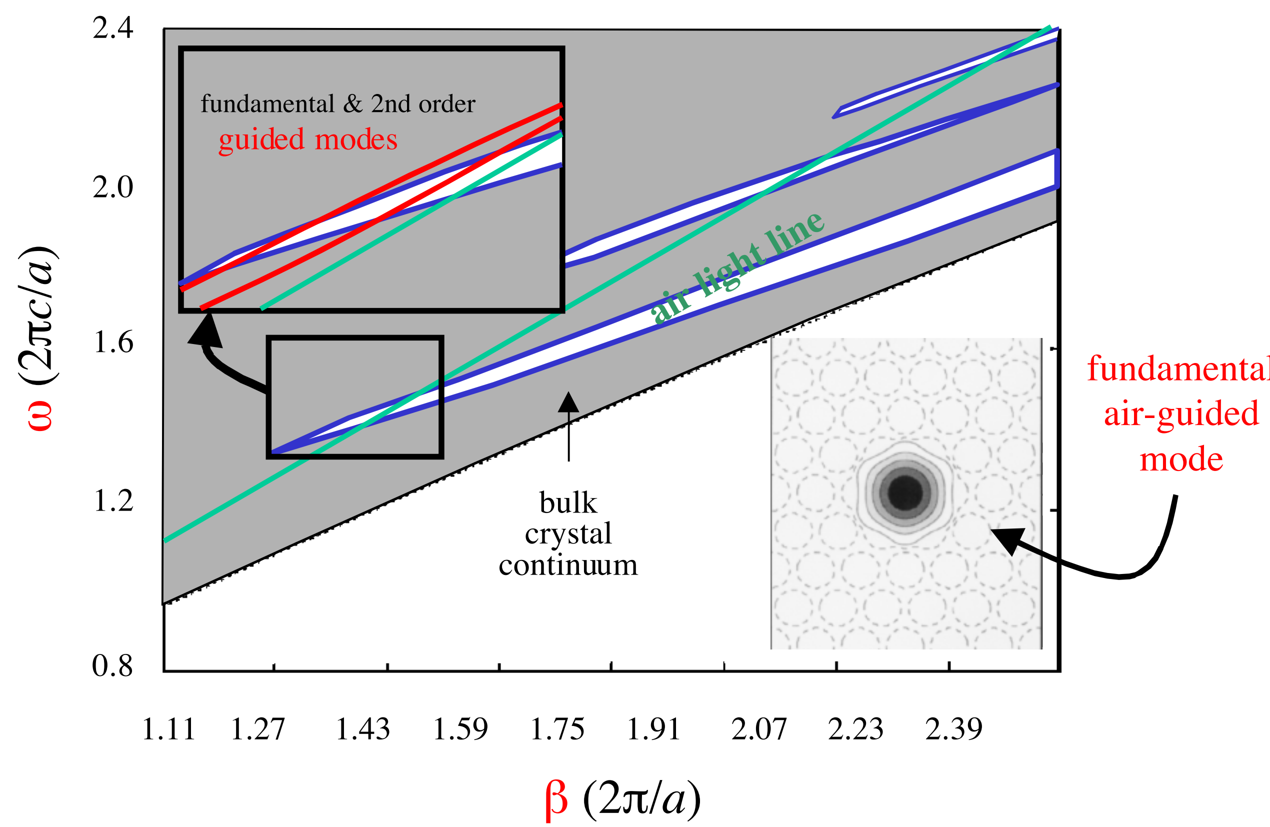


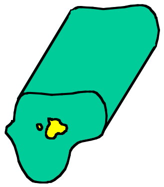
1. Boundary conditions
2. Leakage (finite-size) radiation loss
3. Interior eigenvalues

Gap-guided modes lie above continuum
(~ *N* states for *N*-hole cell)

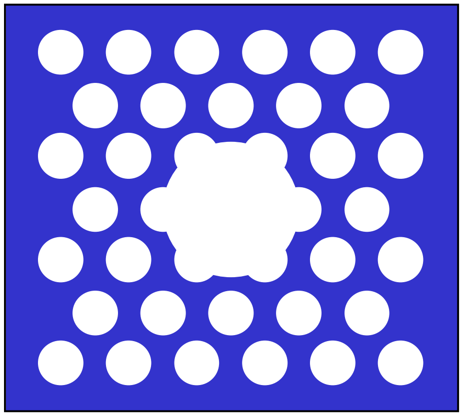
…but most methods compute smallest ω
(or largest β)

[J. Broeng *et al.*, *Opt. Lett.* **25**, 96 (2000)]





Computing Guided (Core) Modes



Gap-guided modes
lie above continuum
(~ N states for N-hole cell)

…but most methods
compute smallest ω
(or largest β)

1. Boundary conditions
2. Leakage (finite-size) radiation loss
3. Interior (of the spectrum) eigenvalues

i. Compute N lowest states first: deflation (orthogonalize to get higher states)

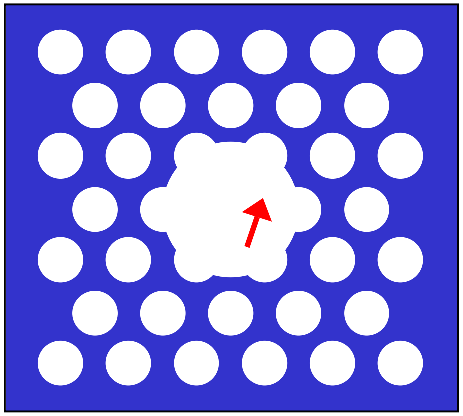
[see previous slide]

ii. Use interior eigensolver method—
…closest eigenvalues to ω_0 (mid-gap)
Jacobi-Davidson,
Arnoldi with shift-and-invert,
smallest eigenvalues of $(A-\omega_0^2)^2$
… convergence often slower

iii. Other methods: FDTD, etc…

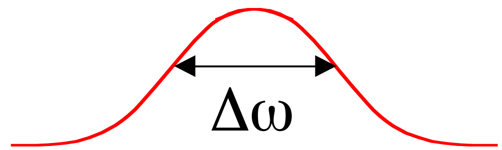
Interior Eigenvalues by FDTD

finite-difference time-domain



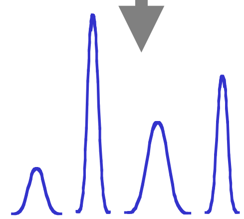
Simulate Maxwell's equations on a discrete grid,
+ PML boundaries + $e^{i\beta z}$ z-dependence

- Excite with broad-spectrum dipole (↑) source



signal processing

complex ω_n ← [Mandelshtam, *J. Chem. Phys*. **107**, 6756 (1997)]

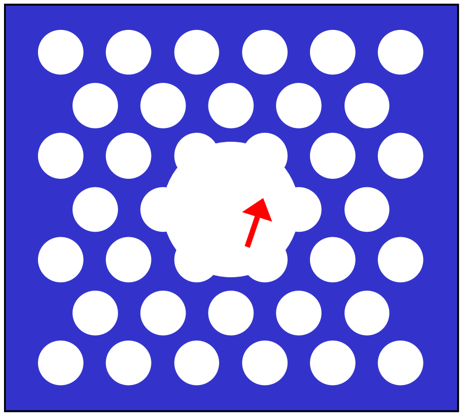


Response is many sharp peaks, one peak per mode

decay rate in time gives loss: $\mathrm{Im}[\beta] = -\,\mathrm{Im}[\omega] \,/\, d\omega/d\beta$

Interior Eigenvalues by FDTD

finite-difference time-domain



Simulate Maxwell's equations on a discrete grid,
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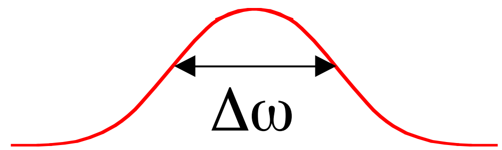
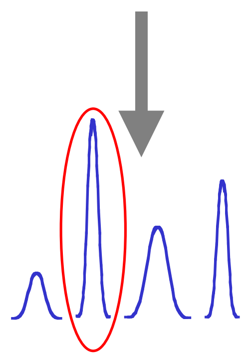


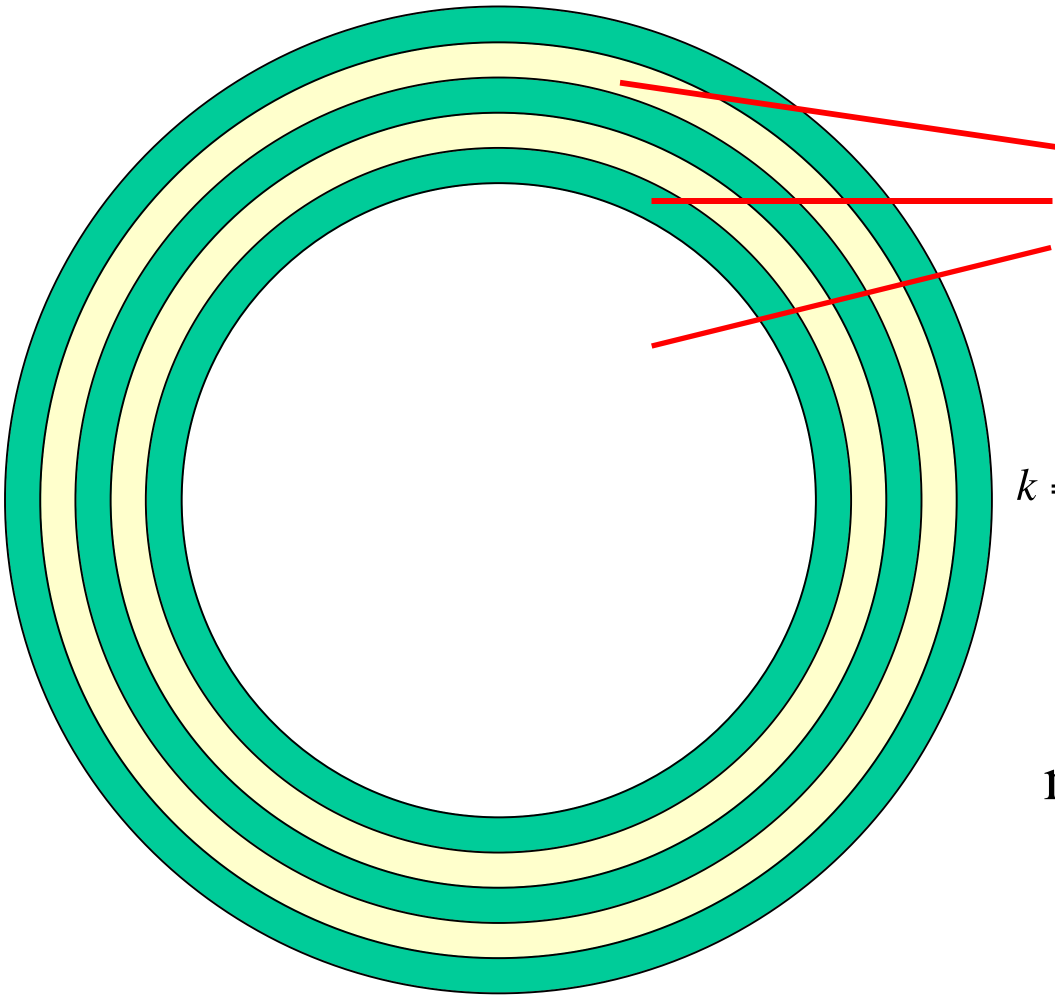


Response is many sharp peaks, one peak per mode

mode field profile

narrow-spectrum source

An Easier Problem: Bragg-fiber Modes



In each concentric region,
solutions are Bessel functions:

$$c\, J_m(kr) + d\, Y_m(kr) \times e^{im\phi}$$

$$k = \sqrt{\left(\frac{\omega}{c}\right)^2 \varepsilon - \beta^2}$$

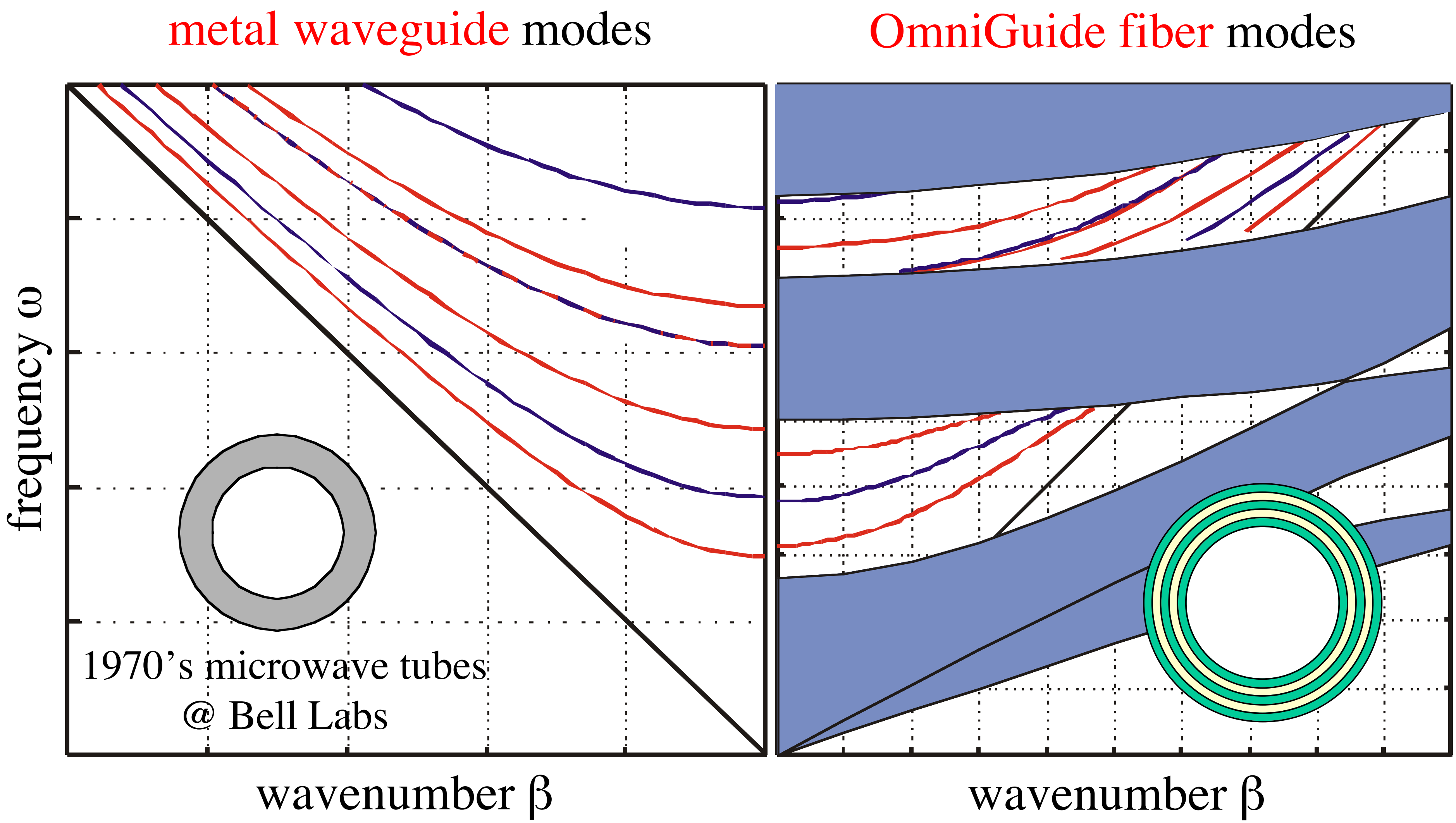
"angular momentum"

At circular interfaces
match boundary conditions
with 4 × 4 transfer matrix

…search for complex β that satisfies: finite at r=0, outgoing at r=∞

[Johnson, *Opt. Express* **9**, 748 (2001)]

Hollow Metal Waveguides, Reborn

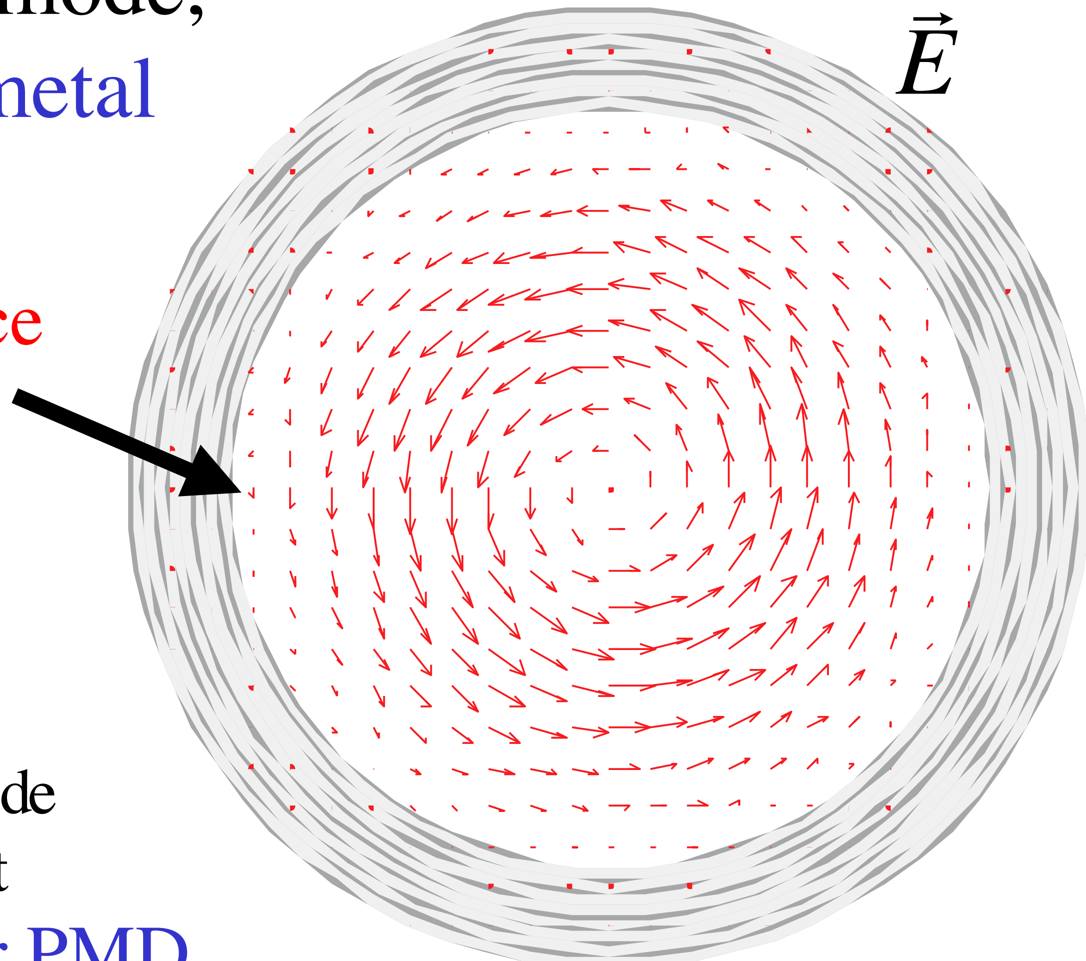


modes are directly analogous to those in hollow metal waveguide

An Old Friend: the TE_{01} mode

lowest-loss mode,
just as in metal

(near) node at interface
= strong confinement
= low losses

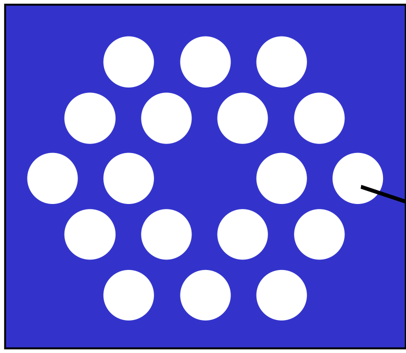


non-degenerate mode
— cannot be split
= no birefringence or PMD

Bushels of Bessels

— A General Multipole Method

[White, *Opt. Express* **9**, 721 (2001)]



only cylinders allowed

Each cylinder has its own Bessel expansion:

$$\text{field} \sim \sum_{m}^{M} c_m J_m + d_m Y_m$$

(m is *not* conserved)

With N cylinders,
get $2NM \times 2NM$ matrix of boundary conditions

Solution gives full complex β,
but takes $O(N^3)$ time

— more than 4–5 periods is difficult

future: "Fast Multipole Method" should reduce to $O(N \log N)$?

Outline

- What are these fibers (and why should I care)?
- The guiding mechanisms: index-guiding and band gaps
- Finding the guided modes
- Small corrections (with big impacts)

All Imperfections are Small

(or the fiber wouldn't work)

- Material absorption: small imaginary $\Delta\varepsilon$
- Nonlinearity: small $\Delta\varepsilon \sim |\mathbf{E}|^2$
- Acircularity (birefringence): small ε boundary shift
- Bends: small $\Delta\varepsilon \sim \Delta x \,/\, R_{\text{bend}}$
- Roughness: small $\Delta\varepsilon$ or boundary shift

Weak effects, long distances: hard to compute directly

— use perturbation theory

Perturbation Theory and Related Methods

(Coupled-Mode Theory, Volume-Current Method, etc.)

Given solution for ideal system
compute approximate effect
of small changes

…solves hard problems starting with easy problems

& provides (semi) analytical insight

Perturbation Theory

for Hermitian eigenproblems

given eigenvectors/values: $\hat{O}|u\rangle = u|u\rangle$

…find change Δu & $\Delta|u\rangle$ for small $\Delta\hat{O}$

Solution:

expand as power series in $\Delta\hat{O}$

$$\Delta u = 0 + \Delta u^{(1)} + \Delta u^{(2)} + \ldots$$

& $\Delta|u\rangle = 0 + \Delta|u\rangle^{(1)} + \ldots$

$$\Delta u^{(1)} = \frac{\langle u|\Delta\hat{O}|u\rangle}{\langle u|u\rangle}$$

(first order is usually enough)

Perturbation Theory

for electromagnetism

$$\Delta\omega^{(1)} = \frac{c^2}{2\omega}\frac{\langle \mathbf{H} | \Delta\hat{A} | \mathbf{H} \rangle}{\langle \mathbf{H} | \mathbf{H} \rangle}$$

$$= -\frac{\omega}{2}\frac{\int \Delta\varepsilon |\mathbf{E}|^2}{\int \varepsilon |\mathbf{E}|^2}$$

…e.g. absorption
gives
imaginary Δω
= decay!

$$\Delta\beta^{(1)} = \Delta\omega^{(1)} / v_g \qquad v_g = \frac{d\omega}{d\beta}$$

A Quantitative Example

…but what about the cladding?

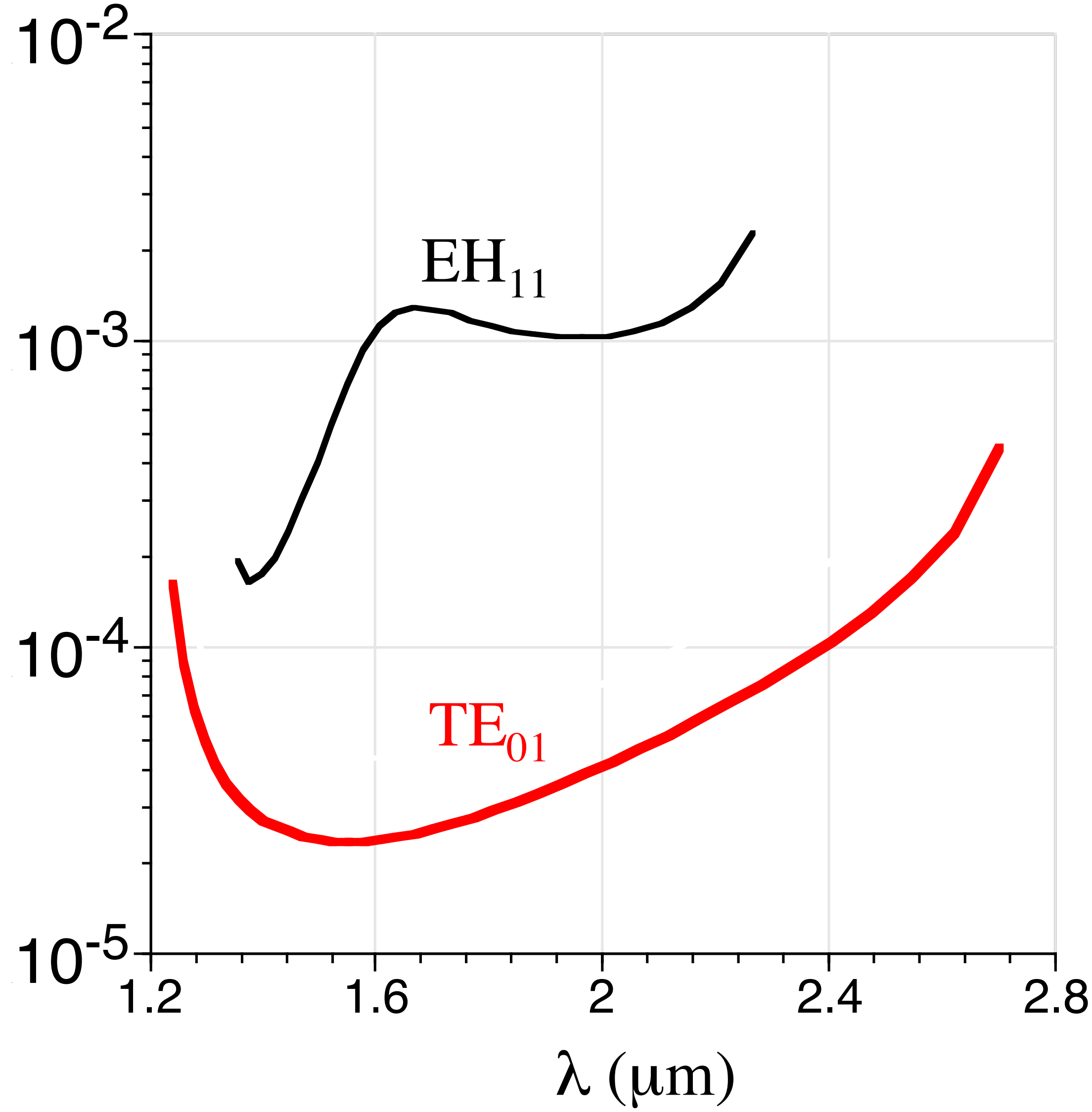
…*some* field penetrates!

& may need to use very "bad" material to get high index contrast

Suppressing Cladding Losses

Material absorption: small imaginary Δε

Mode Losses
÷
Bulk Cladding Losses



Large differential loss

TE_{01} strongly suppresses cladding absorption

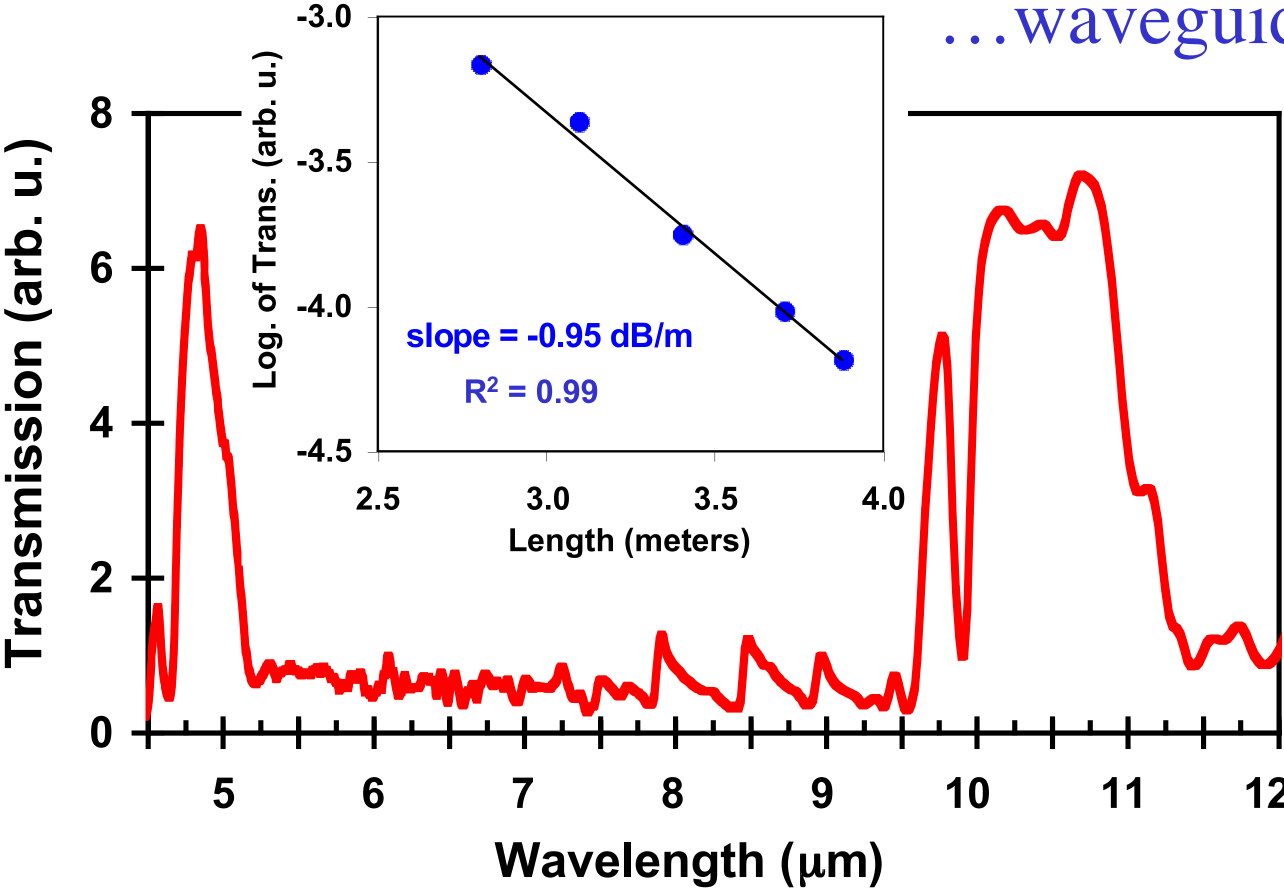
(like ohmic loss, for metal)

High-Power Transmission

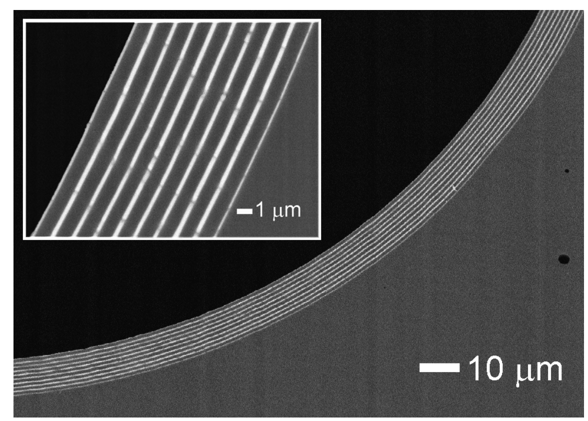
at 10.6μm (no previous dielectric waveguide)

Polymer losses @10.6μm ~ 50,000dB/m…

…waveguide losses ~ 1dB/m



[B. Temelkuran *et al.*, *Nature* **420**, 650 (2002)]



[figs courtesy Y. Fink *et al.*, MIT]

Quantifying Nonlinearity

Kerr nonlinearity: small $\Delta\varepsilon \sim |\mathbf{E}|^2$

$\Delta\beta \sim$ power $P \sim 1$ / lengthscale for nonlinear effects

$$\gamma = \Delta\beta / P$$

= nonlinear-strength parameter determining
self-phase modulation (SPM), four-wave mixing (FWM), …

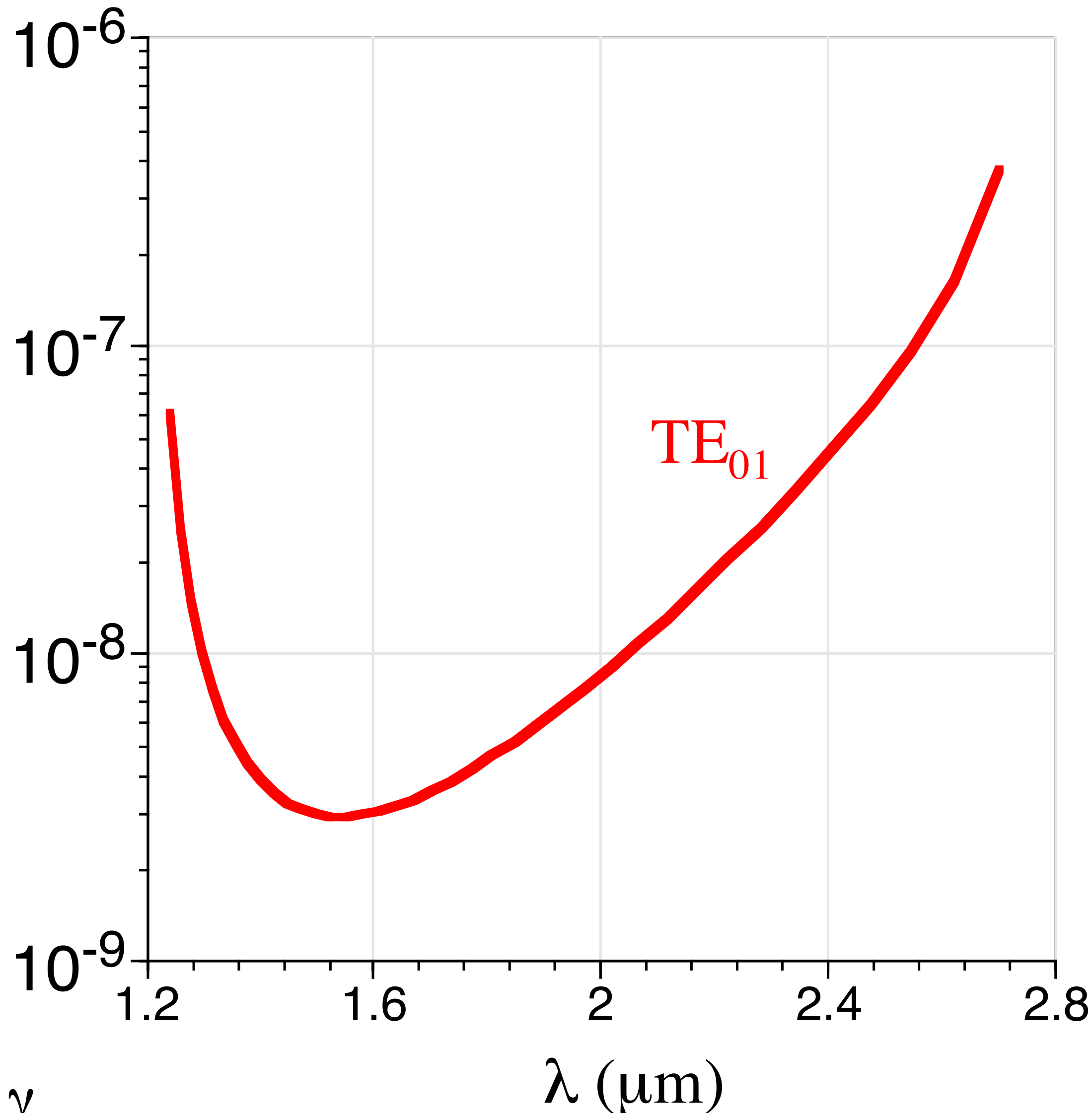
(unlike "effective area,"
tells *where* the field is,
not just how big)

Suppressing Cladding Nonlinearity

Mode Nonlinearity*
÷
Cladding Nonlinearity

Will be dominated by nonlinearity of air

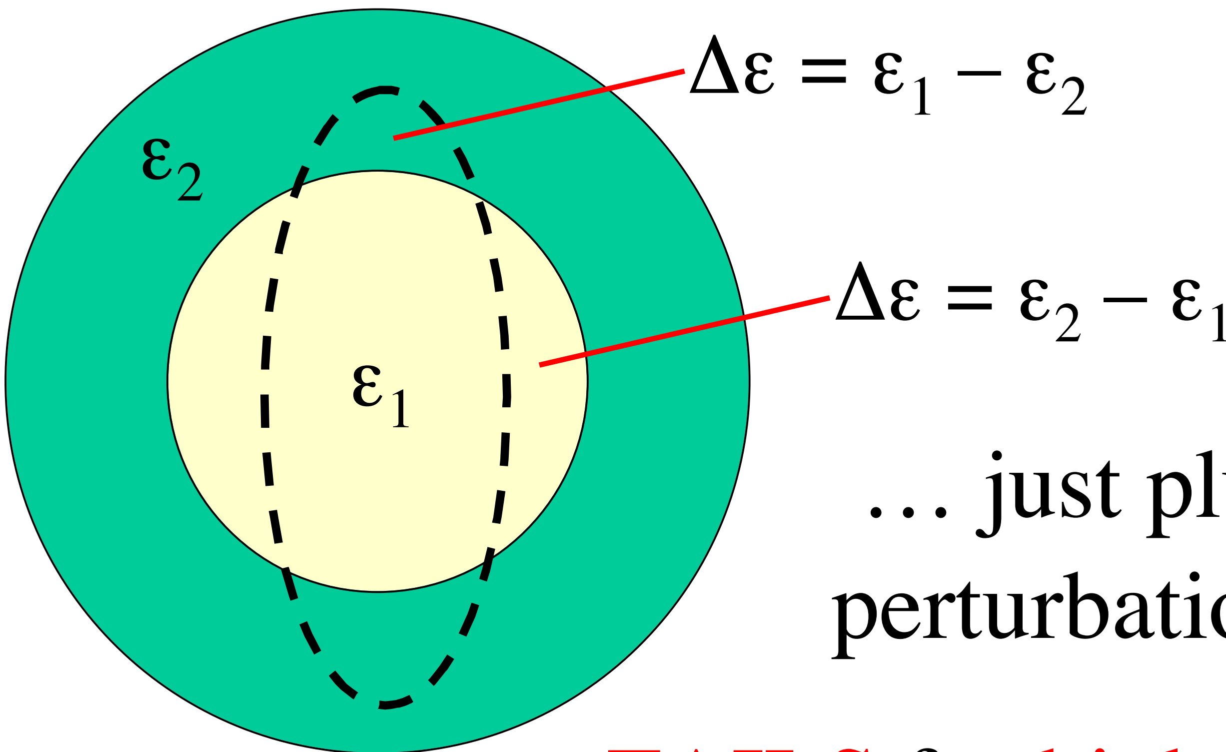
~10,000 times weaker than in silica fiber
(including factor of 10 in area)



* “nonlinearity” = $\Delta\beta^{(1)} / P = \gamma$

Acircularity & Perturbation Theory

(or any shifting-boundary problem)



… just plug Δε's into perturbation formulas?

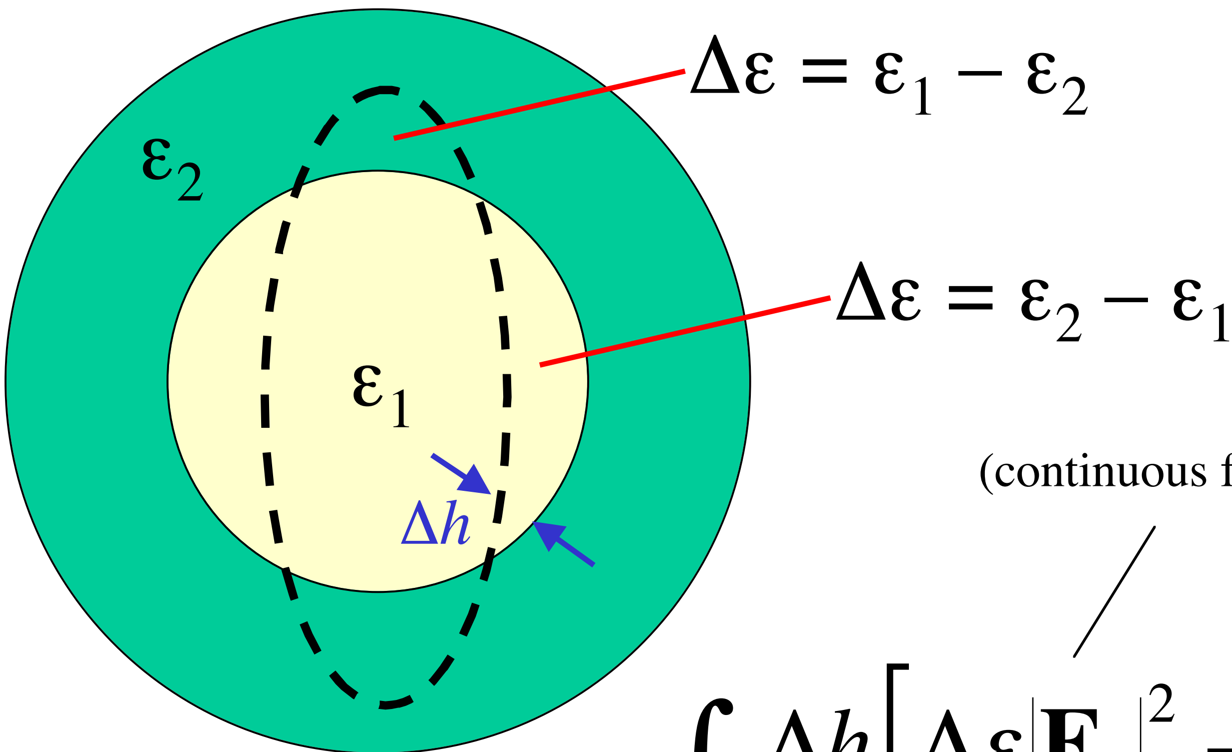
FAILS for high index contrast!

beware field discontinuity…
fortunately, a simple correction exists

[S. G. Johnson *et al.*, *PRE* **65**, 066611 (2002)]

Acircularity & Perturbation Theory

(or any shifting-boundary problem)

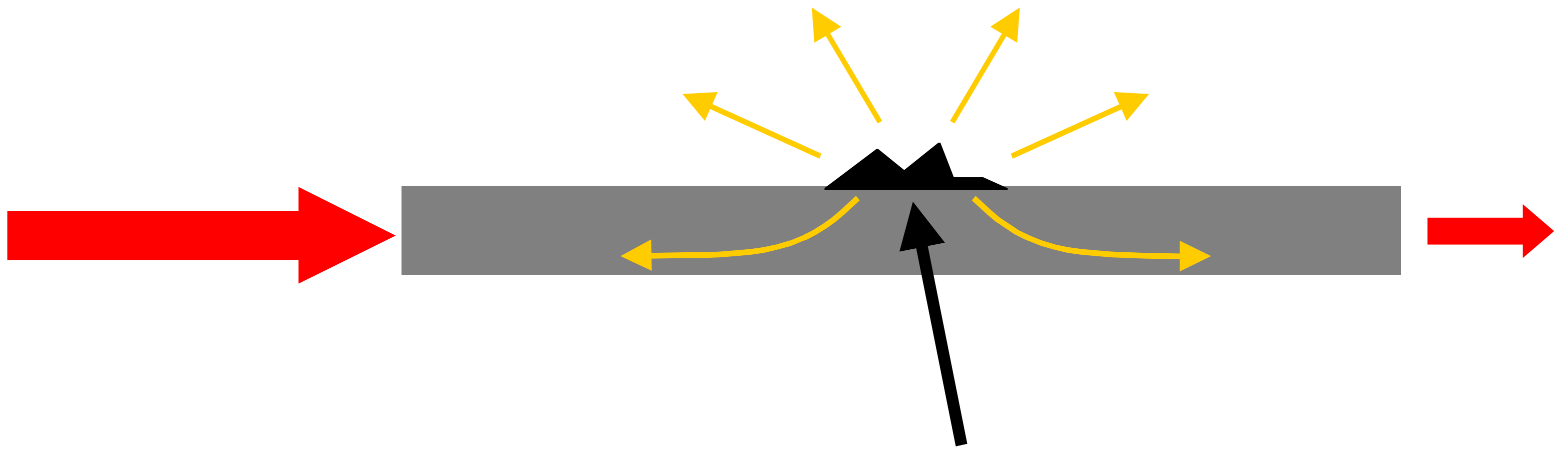


(continuous field components)

$$\Delta\omega^{(1)} = -\frac{\omega}{2} \frac{\int_{\text{surf.}} \Delta h \left[\Delta\varepsilon \left| \mathbf{E}_{\parallel} \right|^2 - \Delta \frac{1}{\varepsilon} \left| D_{\perp} \right|^2 \right]}{\int \varepsilon \left| \mathbf{E} \right|^2}$$

[S. G. Johnson *et al.*, *PRE* **65**, 066611 (2002)]

Loss from Roughness/Disorder



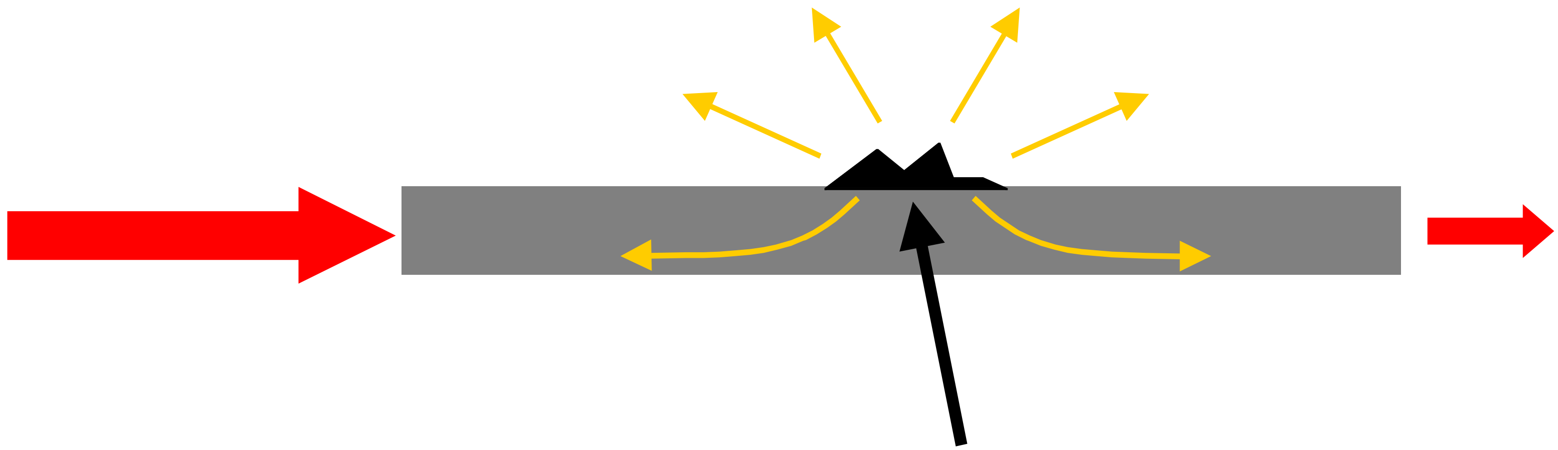
imperfection acts like a volume current

$$\vec{J} \sim \Delta\varepsilon\,\vec{E}_0$$

volume-current method

or Green's functions with first Born approximation

Loss from Roughness/Disorder

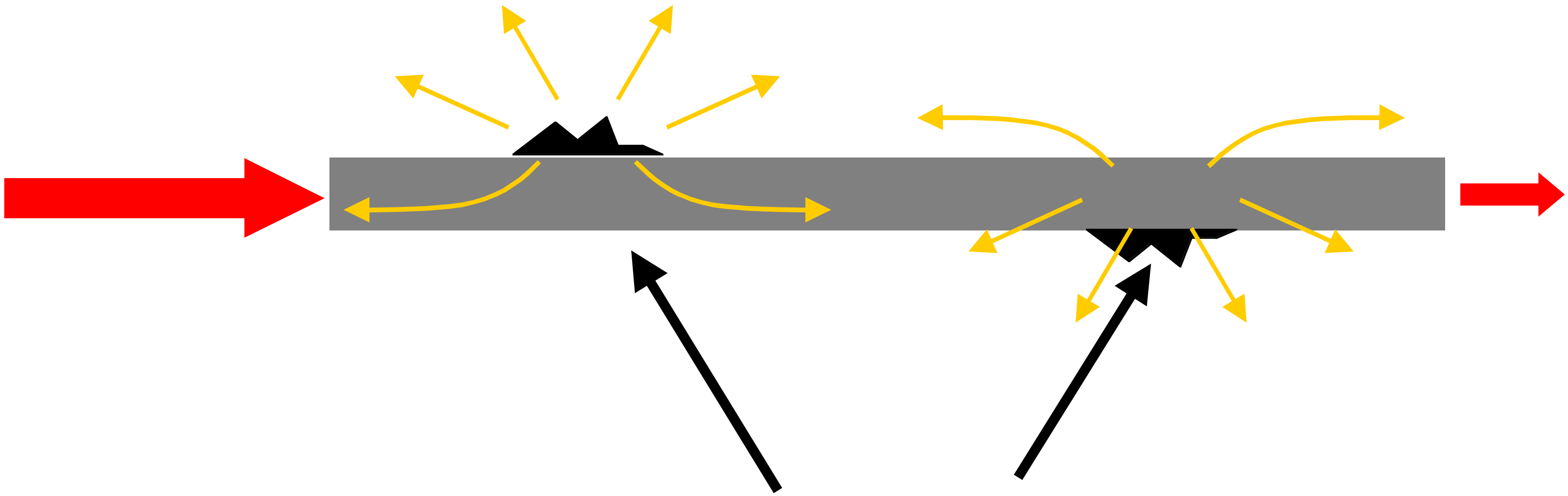


imperfection acts like a volume current

$$\vec{J} \sim \Delta\varepsilon\,\vec{E}_0$$

For surface roughness,
including field discontinuities: $\vec{J} \sim \Delta\varepsilon\,\vec{E}_{\parallel} - \varepsilon\,\Delta\varepsilon^{-1}\,\vec{D}_{\perp}$

Loss from Roughness/Disorder

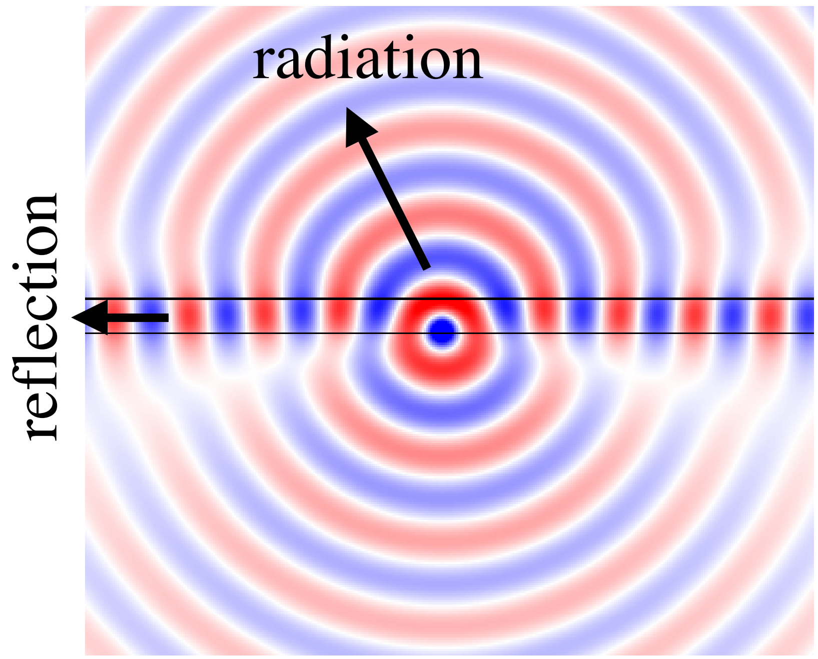


uncorrelated disorder adds *incoherently*

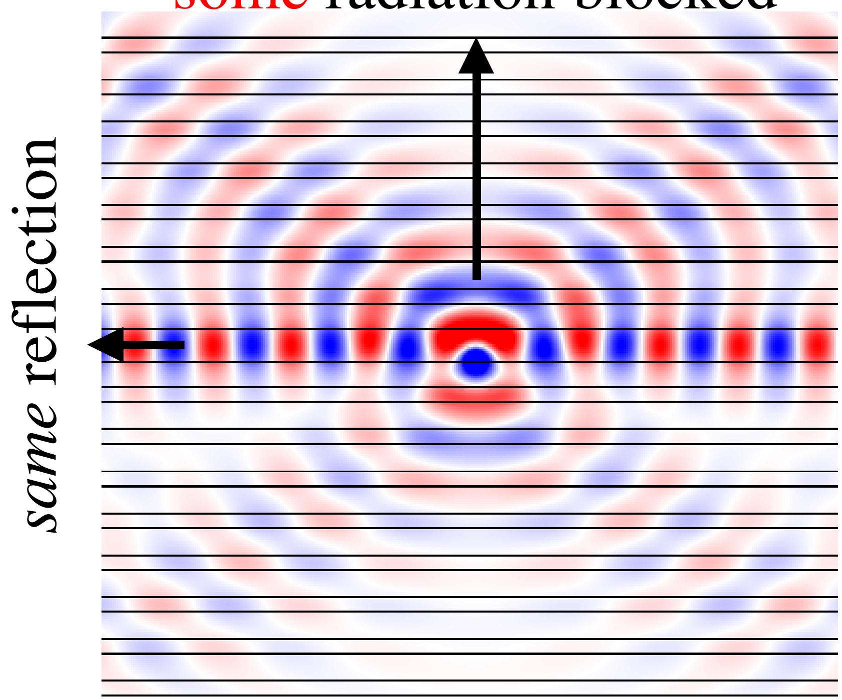
So, compute power P radiated by *one* localized source *J*,
and loss rate ~ P * (mean disorder strength)

Effect of an *Incomplete* Gap

on uncorrelated surface roughness loss



Conventional waveguide
(matching modal area)



…with Si/SiO_2 Bragg mirrors (1D gap)
50% lower losses (in dB)
same reflection

Considerations for Roughness Loss

- Band gap can suppress some radiation
  — typically by at most ~ 1/2, depending on crystal

- Loss ~ $\Delta\varepsilon^2$ ~ 1000 times larger than for silica

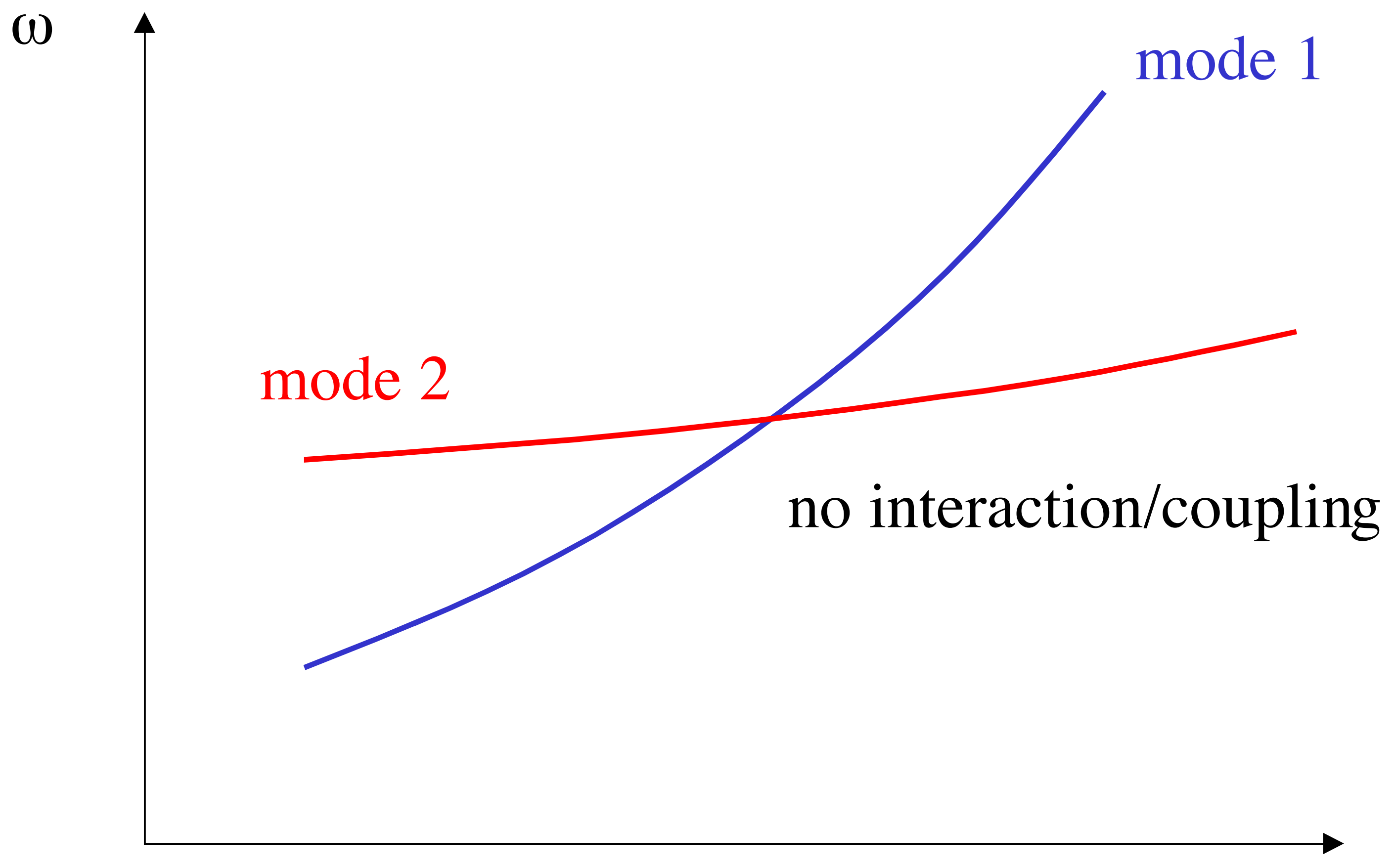
- Loss ~ fraction of $|\mathbf{E}|^2$ in solid material
  — factor of ~ 1/5 for 7-hole PCF
  — ~ 10^{-5} for large-core Bragg-fiber design

- Hardest part is to get reliable statistics for disorder.

Using perturbations to design *big* effects

Perturbation Theory and Dispersion

when two distinct modes cross & interact,
unusual dispersion is produced



Perturbation Theory and Dispersion

when two distinct modes cross & interact,

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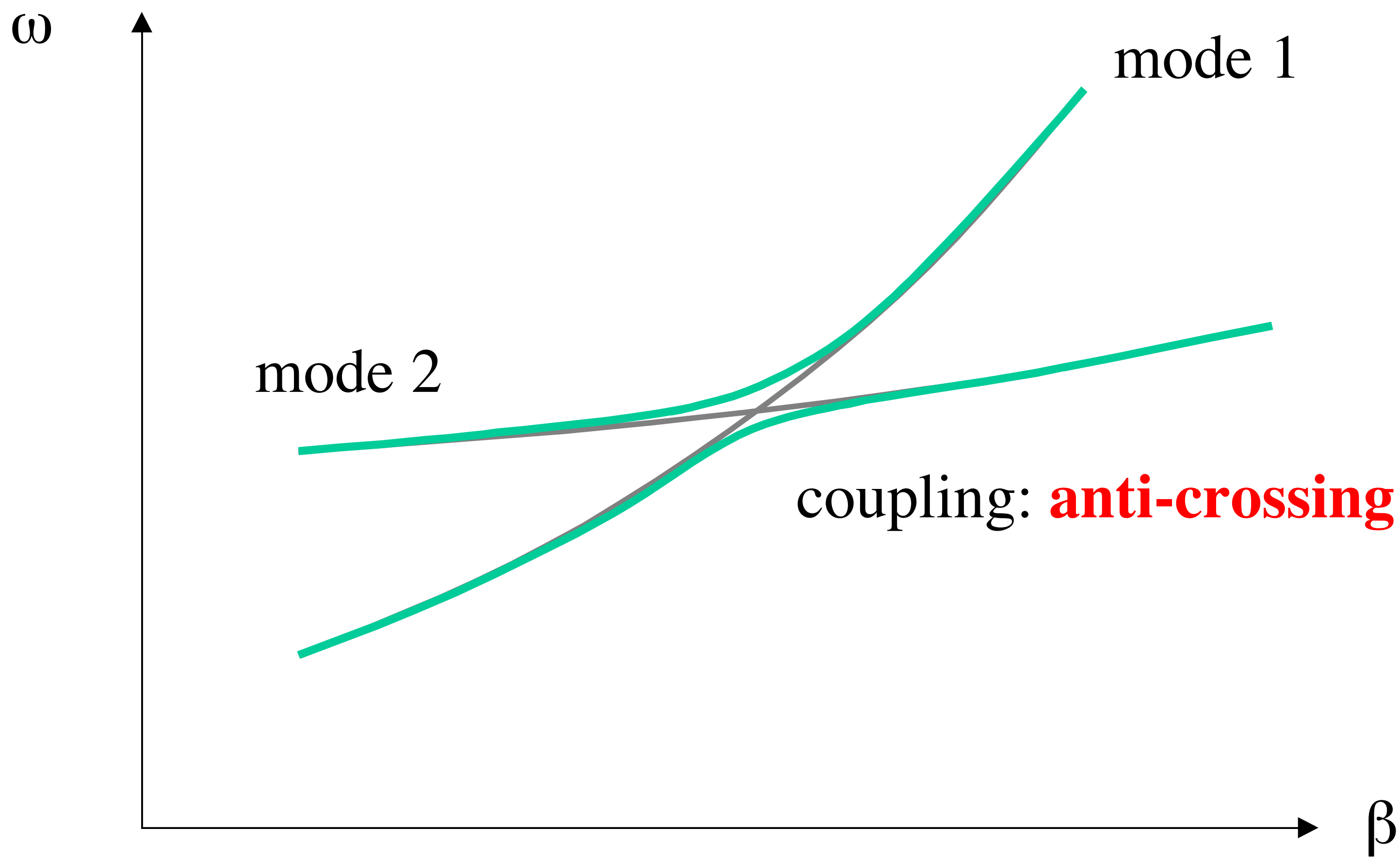
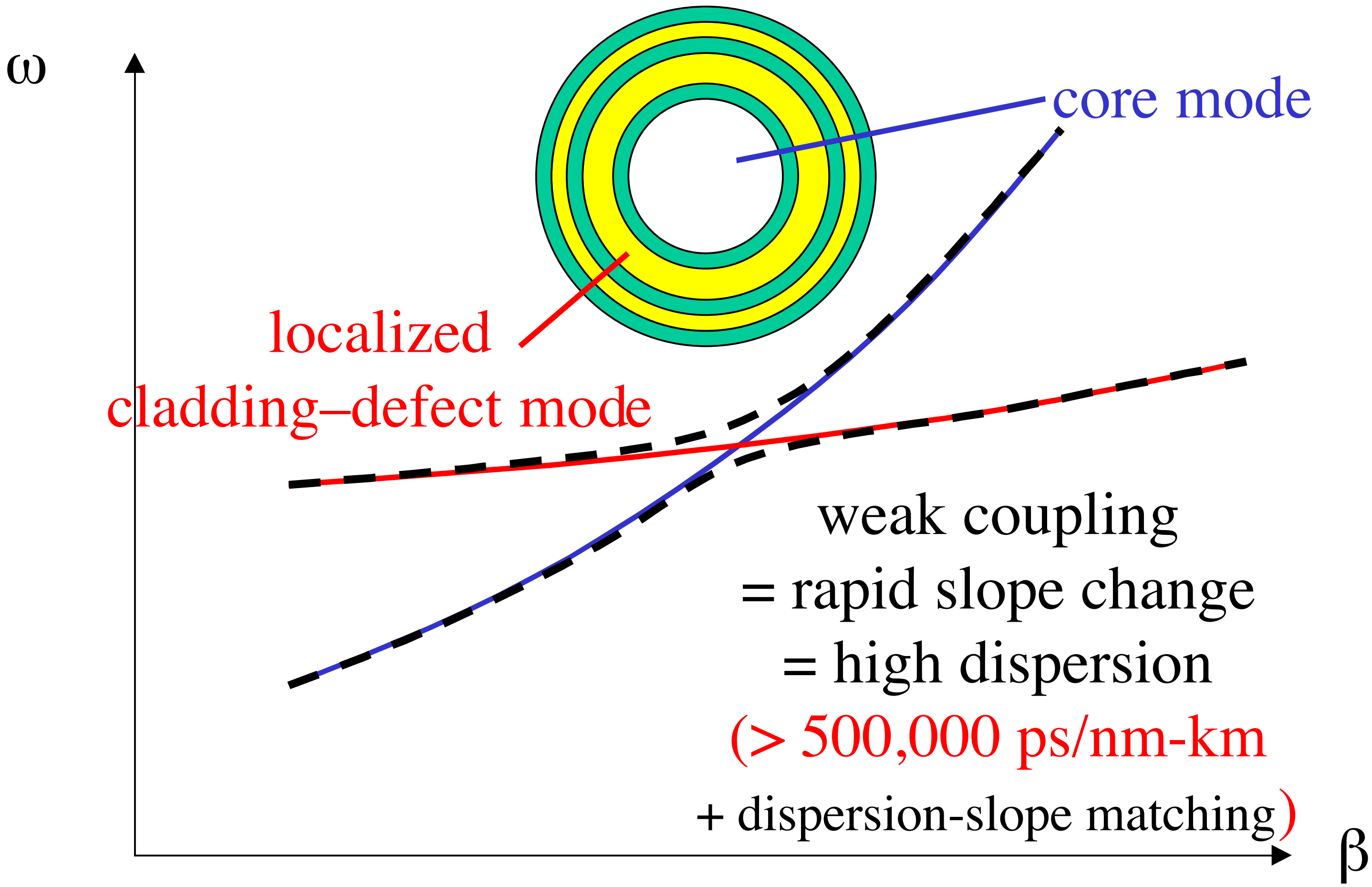
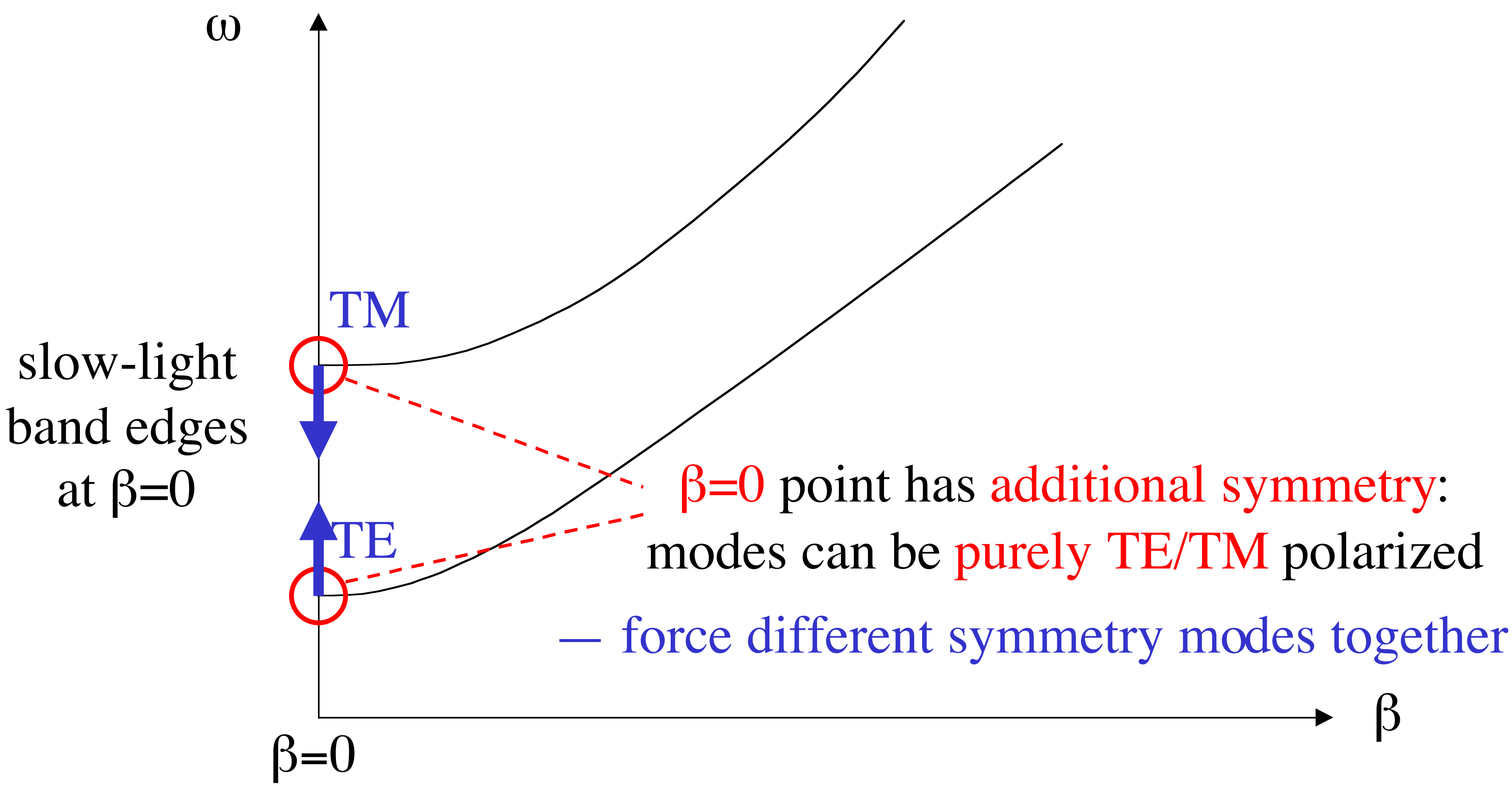


Two Localized Modes
= Very Strong Dispersion



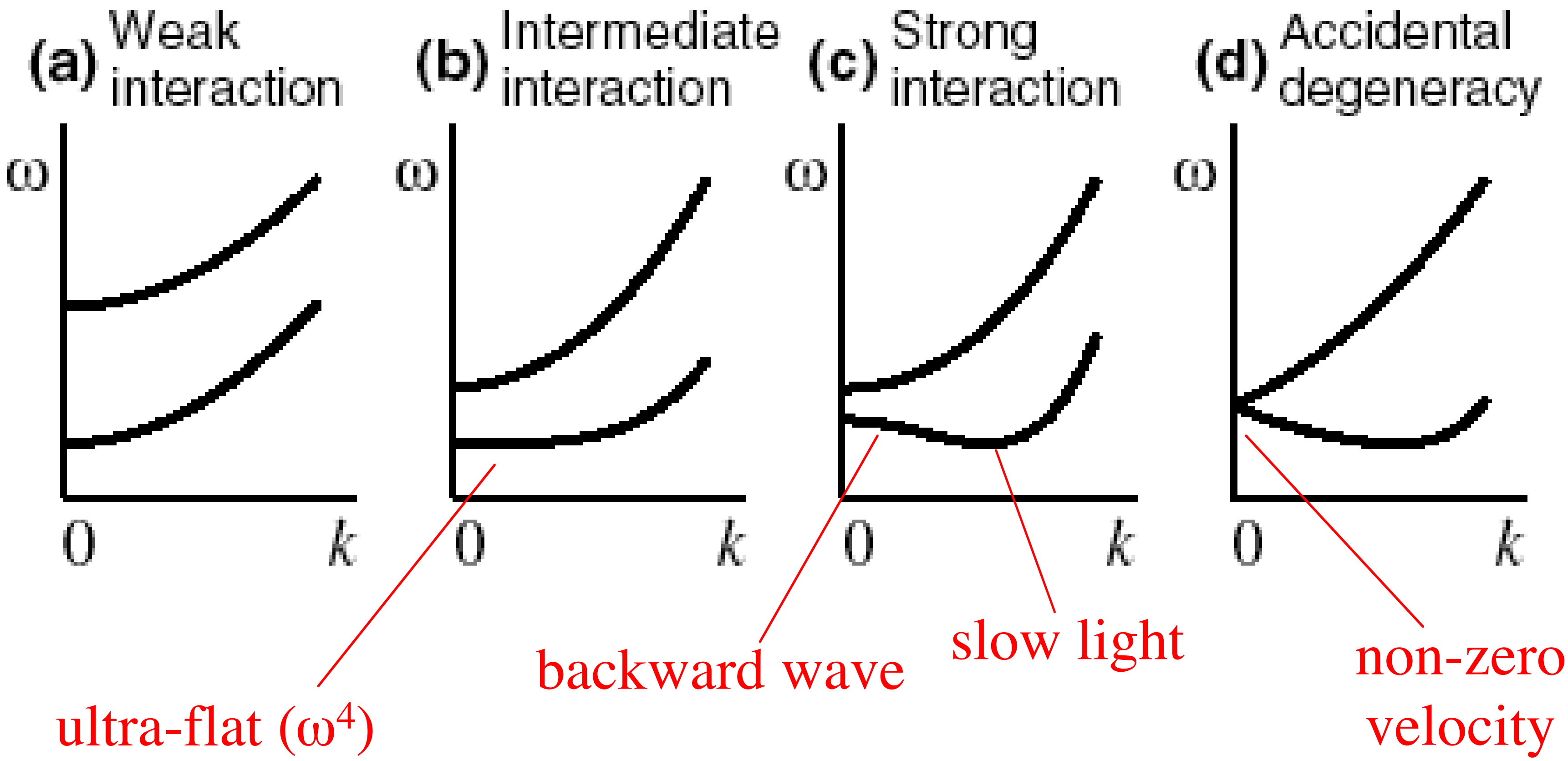
[T. Engeness *et al.*, *Opt. Express* **11**, 1175 (2003)]

(Different-Symmetry) Slow-light Modes = Anomalous Dispersion



[M. Ibanescu *et al.*, *Phys*. *Rev*. *Lett*. **92**, 063903 (2004)]

(Different-Symmetry) Slow-light Modes = Anomalous Dispersion



[M. Ibanescu *et al*., *Phys*. *Rev*. *Lett*. **92**, 063903 (2004)]

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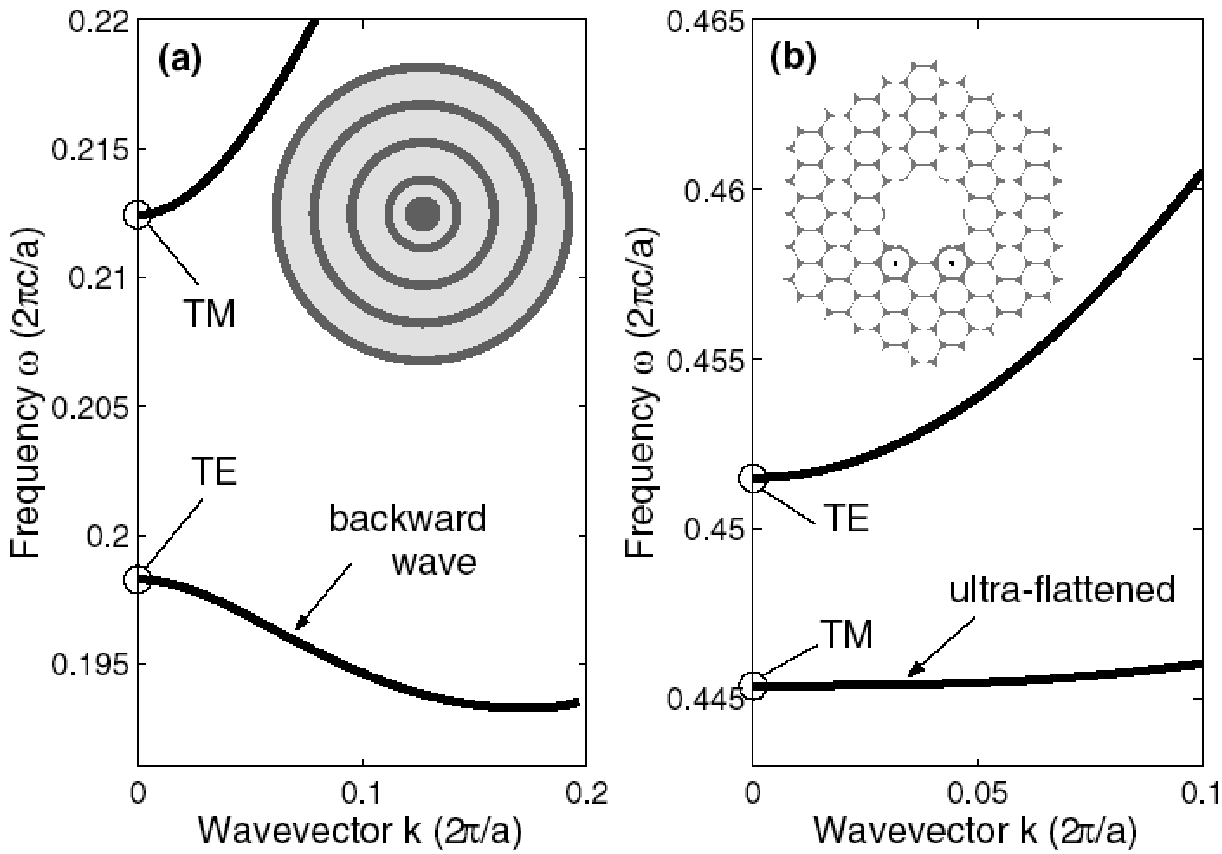

Uses gap at β=0:

perfect metal [1960]

or Bragg fiber

or high-index PCF (n > 2.5)

[M. Ibanescu *et al*., *Phys*. *Rev*. *Lett*. **92**, 063903 (2004)]

Further Reading

Reviews:

• J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals: Molding the Flow of Light* (Princeton Univ. Press, 1995).

• P. Russell, “Photonic-crystal fibers,” *Science* **299**, 358 (2003).

This Presentation, Free Software, Other Material:

`http://ab-initio.mit.edu/photons/tutorial`