0.1 Poincare sphere

We have investigated the formalism of states and density operators for the spin-$\frac{1}{2}$ systems. The key result was the concept of the so called "Bloch Sphere" (or "Ball" for the case of density operators):

- Geometry of the state-space, basis
- Measurement of the spin in x, y, z directions, expectation values
- Coherent superposition, Ensembles, Mixing of several ensembles

Translate these concepts to the case of polarization of the light. Discuss separately the case of continuous regime (intensity of the light) and photon-counting regime (number of detections). Explain, how can we demonstrate some results with polarizing filters, polarization splitting crystall and phase shifters.

0.2 State discrimination

In a two dimensional Hilbert space with basis kets $|x\rangle, |y\rangle$ let $|\Psi\rangle = 1/sqrt2(|x\rangle + |y\rangle)$.

$$\varrho_1 = |\Psi\rangle\langle\Psi|, \qquad \varrho_2 = \frac{1}{2}(|x\rangle\langle x| + |y\rangle\langle y|)$$

Find a standard measurement by which ϱ_1 can sometimes be distinguished from ϱ_2 with absolute certainty.

0.3 Rabi Oscilations

Suppose the total Hamiltonian for a spin-$\frac{1}{2}$ particle is

$$\mathbf{H} = -\gamma\left(B_0\mathbf{S_z} + b(\cos\omega t\mathbf{S_x} + \sin\omega t\mathbf{S_y})\right)$$

which includes a static field B_0 in z direction plus a rotating field in the xy plane. Let the state of the particle be written as

$$|\Psi(t)\rangle = a(t)|+_z\rangle + b(t)|-_z\rangle$$

with initial condition $a(0) = 0, b(0) = 1$. Show that

$$|a(t)|^2 = \frac{\gamma^2 b^2}{\Delta^2 + \gamma^2 b^2}\sin^2\left(t\frac{\sqrt{\Delta^2 + \gamma^2 b^2}}{2}\right)$$

where $\Delta = -\gamma b - \omega$. This expression is known as Rabi Formula.